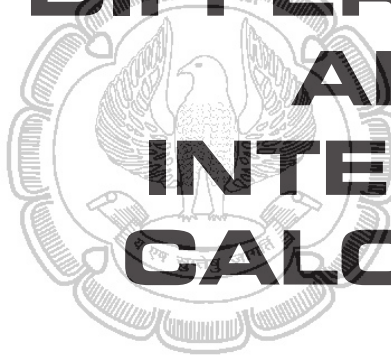


CHAPTER - 9

BASIC CONCEPTS OF DIFFERENTIAL AND INTEGRAL CALCULUS





LEARNING OBJECTIVES

After studying this chapter, you will be able to:

- ◆ Understand the basics of differentiation and integration;
- ◆ Know how to compute derivative of a function by the first principle, derivative of a function by the application of formulae and higher order differentiation;
- ◆ Appreciate the various techniques of integration; and
- ◆ Understand the concept of definite integrals of functions and its properties.

INTRODUCTION TO DIFFERENTIAL AND INTEGRAL CALCULUS (EXCLUDING TRIGONOMETRIC FUNCTIONS)

(A) DIFFERENTIAL CALCULUS

9.A.1 INTRODUCTION

Differentiation is one of the most important fundamental operations in calculus. Its theory primarily depends on the idea of limit and continuity of function.

To express the rate of change in any function we introduce concept of derivative which involves a very small change in the dependent variable with reference to a very small change in independent variable.

Thus differentiation is the process of finding the derivative of a continuous function. It is defined as the limiting value of the ratio of the change (increment) in the function corresponding to a small change (increment) in the independent variable (argument) as the latter tends to zero.

9.A.2 DERIVATIVE OR DIFFERENTIAL COEFFICIENT

Let $y = f(x)$ be a function. If h (or Δx) be the small increment in x and the corresponding increment in y or $f(x)$ be $\Delta y = f(x+h) - f(x)$ then the derivative of $f(x)$ is defined

$$\begin{aligned} \text{as } & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ i.e.} \\ & = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \end{aligned}$$

This is denoted as $f'(x)$ or dy/dx or $\frac{d}{dx} f(x)$. The derivative of $f(x)$ is also known as differential coefficient of $f(x)$ with respect to x . This process of differentiation is called the first principle (or definition or abinitio).

Note: In the light of above discussion a function $f(x)$ is said to be differentiable at

$$\lim_{h \rightarrow 0} \frac{f(x) - f(c)}{x - c} \text{ at } x = c \text{ if it exists which is called the differential coefficient of } f(x) \text{ at } x = c \text{ and is}$$



denoted by $f'(c)$ or $\left[\frac{dy}{dx}\right]_{x=c}$.

We will now study this with an example.

Consider the function $f(x) = x^2$.

By definition

$$\begin{aligned}\frac{d}{dx}f(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x) = 2x + 0 = 2x\end{aligned}$$

Thus, derivative of $f(x)$ exists for all values of x and equals $2x$ at any point x .

Examples of Differentiations from the 1st principle

- i) $f(x) = c$, c being a constant.

$f(x) = c$ since c is constant we may write $f(x+h) = c$.

So $f(x+h) - f(x) = 0$

$$\text{Hence } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

$$\text{So } \frac{d}{dx}(c) = 0$$

- ii) Let $f(x) = x^n$, then $f(x+h) = (x+h)^n$
let $x+h = t$ or $h = t - x$ and as $h \rightarrow 0$ $t \rightarrow x$

$$\begin{aligned}\text{Now } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\ &= \lim_{t \rightarrow x} \frac{(t^n - x^n)}{(t - x)} = nx^{n-1}\end{aligned}$$

$$\text{Hence } \frac{d}{dx}(x^n) = nx^{n-1}$$

- iii) $f(x) = e^x \therefore f(x+h) = e^{x+h}$

$$\begin{aligned}\text{So } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x(e^h - 1)}{h}\end{aligned}$$



$$= e^x \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = e^x \cdot 1$$

Hence $\frac{d}{dx} (e^x) = e^x$

iv) Let $f(x) = a^x$ then $f(x+h) = a^{x+h}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \rightarrow 0} \left[\frac{a^x (a^h - 1)}{h} \right]$$

$$= a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h}$$

$$= a^x \log_e a$$

Thus $\frac{d}{dx} (a^x) = a^x \log_e a$

v) Let $f(x) = \sqrt{x}$. Then $f(x+h) = \sqrt{x+h}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Thus $\frac{d}{dx} (\sqrt{x}) = \frac{1}{2\sqrt{x}}$

vi) $f(x) = \log x \therefore f(x+h) = \log (x+h)$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\log(x+h) - \log x}{h}$$



$$= \lim_{h \rightarrow 0} \frac{\log\left(\frac{x+h}{x}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left\{ \log\left(1 + \frac{h}{x}\right) \right\}$$

Let $\frac{h}{x} = t$ i.e. $h=tx$ and as $h \rightarrow 0$, $t \rightarrow 0$

$$\therefore f'(x) = \lim_{t \rightarrow 0} \frac{1}{tx} \log(1+t) = \frac{1}{x} \lim_{t \rightarrow 0} \frac{1}{t} \log(1+t) = \frac{1}{x} \times 1 = \frac{1}{x}, \text{ since } \lim_{t \rightarrow 0} \frac{\log(1+t)}{t} = 1$$

$$\text{Thus } \frac{d}{dx} (\log x) = \frac{1}{x}$$

9.A.3 SOME STANDARD RESULTS (FORMULAS)

$$(1) \frac{d}{dx} (x^n) = nx^{n-1} \quad (2) \frac{d}{dx} (e^x) = e^x \quad (3) \frac{d}{dx} (a^x) = a^x \log_e a$$

$$(4) \frac{d}{dx} (\text{constant}) = 0 \quad (5) \frac{d}{dx} (e^{ax}) = ae^{ax} \quad (5) \frac{d}{dx} (\log x) = \frac{1}{x}$$

Note: $\frac{d}{dx} \{ c f(x) \} = cf'(x)$ c being constant.

In brief we may write below the above functions and their derivatives:

Table: Few functions and their derivatives

Function	Derivative of the function
$f(x)$	$f'(x)$
x^n	$n x^{n-1}$
e^{ax}	ae^{ax}
$\log x$	$1/x$
a^x	$a^x \log_e a$
c (a constant)	0



We also tabulate the basic laws of differentiation.

Table: Basic Laws for Differentiation

Function	Derivative of the function
(i) $h(x) = c.f(x)$ where c is any real constant (Scalar multiple of a function)	$\frac{d}{dx}\{h(x)\} = c \cdot \frac{d}{dx}\{f(x)\}$
(ii) $h(x) = f(x) \pm g(x)$ (Sum/Difference of function)	$\frac{d}{dx}\{h(x)\} = \frac{d}{dx}\{f(x)\} \pm \frac{d}{dx}\{g(x)\}$
(iii) $h(x) = f(x) \cdot g(x)$ (Product of functions)	$\frac{d}{dx}\{h(x)\} = f(x) \frac{d}{dx}\{g(x)\} + g(x) \frac{d}{dx}\{f(x)\}$
(iv) $h(x) = \frac{f(x)}{g(x)}$ (quotient of function)	$\frac{d}{dx}\{h(x)\} = \frac{g(x) \frac{d}{dx}\{f(x)\} - f(x) \frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$
(v) $h(x) = f\{g(x)\}$	$\frac{d}{dx}\{h(x)\} = \frac{d}{dz} f(z) \cdot \frac{dz}{dx}$, where $z = g(x)$

It should be noted here even though in (ii) (iii) (iv) and (v) we have considered two functions f and g it can be extended to more than two functions by taking two by two.

Example: Differentiate each of the following functions with respect to x :

- (a) $3x^2 + 5x - 2$ (b) $a^x + x^a + a^a$ (c) $\frac{1}{3}x^3 - 5x^2 + 6x - 2\log x + 3$
- (d) $e^x \log x$ (e) $2^x x^5$ (f) $\frac{x^2}{e^x}$
- (g) $e^x / \log x$ (h) $2^x \cdot \log x$ (i) $\frac{2x}{3x^3 + 7}$

Solution: (a) Let $y = f(x) = 3x^2 + 5x - 2$

$$\frac{dy}{dx} = 3 \frac{d}{dx} (x)^2 + 5 \frac{d}{dx} (x) - \frac{d}{dx} (2)$$

$$= 3 \times 2x + 5 \cdot 1 - 0 = 6x + 5$$

(b) Let $h(x) = a^x + x^a + a^a$

$$\frac{d}{dx}\{h(x)\} = \frac{d}{dx} (a^x + x^a + a^a) = \frac{d}{dx} (a^x) + \frac{d}{dx} (x^a) + \frac{d}{dx} (a^a), a^a \text{ is a constant}$$



$$= a^x \log a + ax^{a-1} + 0 = a^x \log a + ax^{a-1}.$$

$$(c) \text{ Let } f(x) = \frac{1}{3}x^3 - 5x^2 + 6x - 2\log x + 3 \quad \therefore \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(\frac{1}{3}x^3 - 5x^2 + 6x - 2\log x + 3\right)$$
$$= \frac{1}{3} \cdot 3x^2 - 5 \cdot 2x + 6 \cdot 1 - 2 \cdot \frac{1}{x} + 0 = x^2 - 10x + 6 - \frac{2}{x}.$$

$$(d) \text{ Let } y = e^x \log x$$

$$\frac{dy}{dx} = e^x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(e^x) \text{ (Product rule)}$$
$$= \frac{e^x}{x} + e^x \log x = \frac{e^x}{x}(1 + x \log x)$$

$$\text{So } \frac{dy}{dx} = \frac{e^x}{x}(1 + x \log x)$$

$$(e) y = 2^x x^5$$

$$\frac{dy}{dx} = x^5 \frac{d}{dx}(2^x) + 2^x \frac{d}{dx}(x^5) \text{ Product Rule}$$
$$= x^5 2^x \log_e 2 + 5 \cdot 2^x x^4$$

$$(f) \text{ let } y = \frac{x^2}{e^x}$$

$$\frac{dy}{dx} = \frac{e^x \frac{d}{dx}(x^2) - x^2 \frac{d}{dx}(e^x)}{(e^x)^2} \text{ (Quotient Rule)}$$

$$= \frac{2xe^x - x^2 e^x}{(e^x)^2} = \frac{x(2-x)}{e^x}$$

$$(g) \text{ Let } y = e^x / \log x$$

$$\text{so } \frac{dy}{dx} = \frac{(\log x) \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\log x)}{(\log x)^2} \text{ (Quotient Rule)}$$

$$= \frac{e^x \log x - e^x/x}{(\log x)^2} = \frac{e^x x \log x - e^x}{x(\log x)^2}$$

$$\text{So } \frac{dy}{dx} = \frac{e^x(x \log x - 1)}{x(\log x)^2}$$



(h) Let $h(x) = 2^x \cdot \log x$

The given function $h(x)$ is appearing here as product of two functions

$$f(x) = 2^x \quad \text{and} \quad g(x) = \log x.$$

$$= \frac{d}{dx} \{h(x)\} = \frac{d}{dx} (2^x \cdot \log x) = 2^x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (2^x).$$

$$2^x \times \frac{1}{x} + \log x \cdot (2^x \log 2) = \frac{2^x}{x} + 2^x \log 2 \log x$$

(i) Let $h(x) = \frac{2x}{3x^3 + 7}$ [Given function appears as the quotient of two functions]

$$f(x) = 2x \quad \text{and} \quad g(x) = 3x^3 + 7$$

$$\frac{d}{dx} \{h(x)\} = \frac{(3x^3 + 7) \frac{d}{dx} (2x) - 2x \frac{d}{dx} (3x^3 + 7)}{(3x^3 + 7)^2} = \frac{(3x^3 + 7) \cdot 2 - 2x \cdot (9x^2 + 0)}{(3x^3 + 7)^2}$$

$$= \frac{2 \{ (3x^3 + 7) - 9x^3 \}}{(3x^3 + 7)^2} = \frac{2(7 - 6x^3)}{(3x^3 + 7)^2}$$

9.A.4 DERIVATIVE OF A FUNCTION OF FUNCTION

If $y = f[h(x)]$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = f'(u) \times h'(x)$ where $u = h(x)$

Example: Differentiate $\log(1 + x^2)$ wrt. x

Solution: Let $y = \log(1 + x^2) = \log t$ when $t = 1 + x^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{1}{t} \times (0 + 2x) = \frac{2x}{t} = \frac{2x}{(1+x^2)}$$

This is an example of derivative of function of a function and the rule is called Chain Rule.

9.A.5 IMPLICIT FUNCTIONS

A function in the form $f(x, y) = 0$ eg. $x^2y^2 + 3xy + y = 0$ where y cannot be directly defined as a function of x is called an implicit function of x .

In case of implicit functions if y be a differentiable function of x no attempt is required to express y as an implicit function of x for finding out $\frac{dy}{dx}$. In such case differentiation of both sides with respect of x and substitution of $\frac{dy}{dx} = y_1$ gives the result. Thereafter y_1 may be obtained by solving the resulting equation.



Example: Find $\frac{dy}{dx}$ for $x^2y^2 + 3xy + y = 0$

Solution: $x^2y^2 + 3xy + y = 0$

Differentiating with respect to x we see

$$x^2 \frac{d}{dx} (y^2) + y^2 \frac{d}{dx} (x^2) + 3x \frac{d(y)}{dx} y + 3y \frac{d}{dx} (x) + \frac{dy}{dx} = 0$$

$$\text{or } 2yx^2 \frac{dy}{dx} + 2xy^2 + 3x \frac{dy}{dx} + 3y \frac{d(x)}{dx} + \frac{dy}{dx} = 0, \frac{d}{dx}(x)=1, \frac{d(y^2)}{dx} = 2y \frac{dy}{dx} \text{ (chain rule)}$$

$$\text{or } (2yx^2 + 3x + 1) \frac{dy}{dx} + 2xy^2 + 3y = 0$$

$$\text{or } \frac{dy}{dx} = - \frac{(2xy^2 + 3y)}{(2x^2y + 3x + 1)}$$

This is the procedure for differentiation of Implicit Function.

9.A.6 PARAMETRIC EQUATION

When both the variables are expressed in terms of a parameter (a third variable) the involved equations are called parametric equations.

For the parametric equations $x = f(t)$ and $y = h(t)$ the differential coefficient $\frac{dy}{dx}$

is obtained by using $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

Example: Find $\frac{dy}{dx}$ if $x = at^3$, $y = a / t^3$

Solution : $\frac{dx}{dt} = 3at^2$; $\frac{dy}{dt} = -3a / t^4$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-3a}{t^4} \times \frac{1}{3at^2} = \frac{-1}{t^6}$$

This is the procedure for differentiation of parametric functions.

9.A.7 LOGARITHMIC DIFFERENTIATION

The process of finding out derivative by taking logarithm in the first instance is called logarithmic differentiation. The procedure is convenient to adopt when the function to be differentiated involves a function in its power or when the function is the product of number



of functions.

Example: Differentiate x^x w.r.t. x

Solution: let $y = x^x$

Taking logarithm,

$$\log y = x \log x$$

Differentiating with respect to x ,

$$\frac{1}{y} \frac{dy}{dx} = \log x + \frac{x}{x} = 1 + \log x$$

$$\text{or } \frac{dy}{dx} = y (1 + \log x) = x^x (1 + \log x)$$

This procedure is called logarithmic differentiation.

9.A.8 SOME MORE EXAMPLES

- (1) If $y = \sqrt{\frac{1-x}{1+x}}$ show that $(1-x^2) \frac{dy}{dx} + y = 0$.

Solution: Taking logarithm, we may write $\log y = \frac{1}{2} \{\log(1-x) - \log(1+x)\}$

Differentiating throughout we have

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \frac{d}{dx} \{\log(1-x) - \log(1+x)\} = \frac{1}{2} \left(\frac{-1}{1-x} - \frac{1}{1+x} \right) = -\frac{1}{1-x^2}$$

$$\text{By cross-multiplication } (1-x^2) \frac{dy}{dx} = -y$$

Transposing $(1-x^2) \frac{dy}{dx} + y = 0$.

- (2) Differentiate the following w.r.t. x :

(a) $\log(x + \sqrt{x^2 + a^2})$

(b) $\log(\sqrt{x-a} + \sqrt{x-b})$.

Solution: (a) $y = \log(x + \sqrt{x^2 + a^2})$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{(x + \sqrt{x^2 + a^2})} \left(1 + \frac{1}{2\sqrt{x^2 + a^2}}(2x) \right) \\ &= \frac{1}{(x + \sqrt{x^2 + a^2})} + \frac{x}{(x + \sqrt{x^2 + a^2})\sqrt{x^2 + a^2}} \end{aligned}$$



$$= \frac{(x + \sqrt{x^2 + a^2})}{(x + \sqrt{x^2 + a^2}) \sqrt{x^2 + a^2}} = \frac{1}{\sqrt{x^2 + a^2}}$$

(b) Let $y = \log(\sqrt{x-a} + \sqrt{x-b})$

$$\begin{aligned} \text{or } \frac{dy}{dx} &= \frac{1}{\sqrt{x-a} + \sqrt{x-b}} \left(\frac{1}{2\sqrt{x-a}} + \frac{1}{2\sqrt{x-b}} \right) = \frac{(\sqrt{x-b} + \sqrt{x-a})}{(\sqrt{x-a} + \sqrt{x-b}) 2\sqrt{x-a}\sqrt{x-b}} \\ &= \frac{1}{2\sqrt{x-a}\sqrt{x-b}} \end{aligned}$$

(3) If $x^m y^n = (x+y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$

Solution : $x^m y^n = (x+y)^{m+n}$

Taking log on both sides

$$\begin{aligned} \log x^m y^n &= (m+n) \log(x+y) \\ \text{or } m \log x + n \log y &= (m+n) \log(x+y) \end{aligned}$$

$$\text{so } \frac{m}{x} + \frac{n}{y} \frac{dy}{dx} = \frac{(m+n)}{(x+y)} \left(1 + \frac{dy}{dx} \right)$$

$$\text{or } \left(\frac{n}{y} - \frac{m+n}{x+y} \right) \frac{dy}{dx} = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\text{or } \frac{(nx+ny-my-ny)}{y(x+y)} \frac{dy}{dx} = \frac{mx+nx-mx-my}{x(x+y)}$$

$$\text{or } \frac{(nx-my)}{y} \frac{dy}{dx} = \frac{nx-my}{x}$$

$$\text{or } \frac{dy}{dx} = \frac{y}{x} \text{ Proved.}$$

(4) If $x^y = e^{x-y}$ Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Solution : $x^y = e^{x-y}$

$$\text{So } y \log x = (x-y) \log e$$

$$\text{or } y \log x = (x-y) \dots\dots\dots(a)$$

Differentiating w.r.t. x we get

$$\frac{y}{x} + \log x \frac{dy}{dx} = 1 - \frac{dy}{dx}$$



or $(1 + \log x) \frac{dy}{dx} = 1 - \frac{y}{x}$

or $\frac{dy}{dx} = \frac{(x-y)}{x(1+\log x)}$, substituting $x-y = \log x$, from (a) we have

or $\frac{dy}{dx} = \frac{y(\log x)}{x(1+\log x)}$ (b)

From (a) $y(1 + \log x) = x$

or $\frac{y}{x} = \frac{1}{(1+\log x)}$

From (b) $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

9.A.9 BASIC IDEA ABOUT HIGHER ORDER DIFFERENTIATION

Let $y = f(x) = x^4 + 5x^3 + 2x^2 + 9$

$\frac{dy}{dx} = \frac{d}{dx} f(x) = 4x^3 + 15x^2 + 4x = f'(x)$

Since $f(x)$ is a function of x it can be differentiated again.

Thus $\frac{d}{dx} \left(\frac{dy}{dx} \right) = f'' = \frac{d}{dx} (4x^3 + 15x^2 + 4x) = 12x^2 + 30x + 4$

$\frac{d}{dx} \left(\frac{dy}{dx} \right)$ is written as $\frac{d^2y}{dx^2}$ (read as d two y dx two) and is called the second derivative of

y with respect to x while $\frac{dy}{dx}$ is called the first derivative. Again the second derivative here

being a function of x can be differentiated again and $\frac{d}{dx} \left(\frac{d^2y}{dx^2} \right)$
 $= f'''(x) = 24x + 30$.

Example: If $y = ae^{mx} + be^{-mx}$ prove that $\frac{d^2y}{dx^2} = m^2y$.

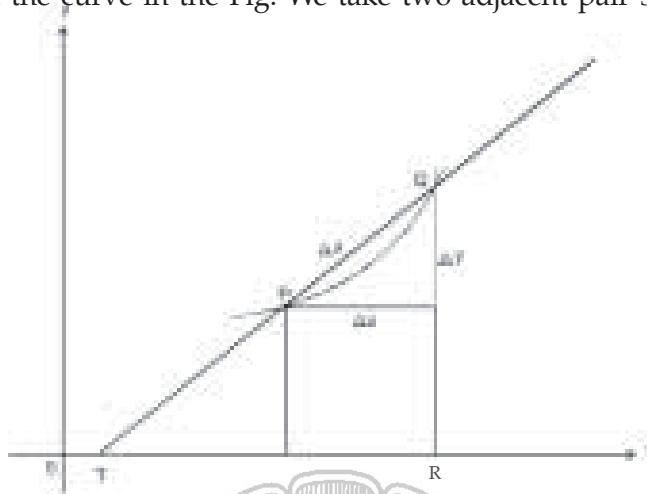
Solution: $\frac{dy}{dx} = \frac{d}{dx} (ae^{mx} + be^{-mx}) = ame^{mx} - bme^{-mx}$

$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (ame^{mx} - bme^{-mx})$
 $= am^2e^{mx} + bm^2e^{-mx} = m^2 (ae^{mx} + be^{-mx}) = m^2y$.



9.A.10 GEOMETRIC INTERPRETATION OF THE DERIVATIVE

Let $f(x)$ represent the curve in the Fig. We take two adjacent pair's P and Q on the curve



Let $f(x)$ represent the curve in the fig. We take two adjacent pair's P and Q on the curve whose coordinates are (x, y) and $(x + \Delta x, y + \Delta y)$ respectively. The slope of the chord TPQ is given by $\Delta y / \Delta x$ when $\Delta x \rightarrow 0$, $Q \rightarrow P$. TPQ becomes the tangent at P and $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

The derivative of $f(x)$ at a point x represents the slope (or sometime called the gradient of the curve) of the tangent to the curve $y = f(x)$ at the point x . If $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$ exists for a particular point say $x = a$ and $f(a)$ is finite we say the function is differentiable at $x = a$ and continuous at that point.

Example : Find the gradient of the curve $y = 3x^2 - 5x + 4$ at the point (12).

Solution : $y = 3x^2 - 5x + 4 \therefore \frac{dy}{dx} = 6x - 5$

$$\text{so } [dy / dx]_{x=12} = 6 \cdot 12 - 5 = 72 - 5 = 67$$

Thus the gradient of the curve at (12) is 67.



Exercise 9 (A)

Choose the most appropriate option (a) (b) (c) or (d)

- The gradient of the curve $y = 2x^3 - 3x^2 - 12x + 8$ at $x = 0$ is
 a) -12 b) 12 c) 0 d) none of these
- The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at $x = 0$ is
 a) 3 b) -3 c) $1/3$ d) none of these
- The derivative of $y = \sqrt{x+1}$ is
 a) $1 / \sqrt{x+1}$ b) $-1 / \sqrt{x+1}$ c) $1 / 2 \sqrt{x+1}$ d) none of these
- If $f(x) = e^{ax^2+bx+c}$ the $f'(x)$ is
 a) e^{ax^2+bx+c} b) $e^{ax^2+bx+c} (2ax + b)$ c) $2ax + b$ d) none of these
- If $f(x) = \frac{x^2+1}{x^2-1}$ then $f'(x)$ is
 a) $-4x / (x^2 - 1)^2$ b) $4x / (x^2 - 1)^2$ c) $x / (x^2 - 1)^2$ d) none of these
- If $y = x(x-1)(x-2)$ then $\frac{dy}{dx}$ is
 a) $3x^2 - 6x + 2$ b) $-6x + 2$ c) $3x^2 + 2$ d) none of these
- The gradient of the curve $y - xy + 2px + 3qy = 0$ at the point $(3, 2)$ is $-\frac{2}{3}$. The values of p and q are
 a) $(1/2, 1/2)$ b) $(2, 2)$ c) $(-1/2, -1/2)$ d) $(1/2, 1/6)$
- The curve $y^2 = ux^3 + v$ passes through the point $P(2, 3)$ and $\frac{dy}{dx} = 4$ at P. The values of u and v are
 a) $(u = 2, v = 7)$ b) $(u = 2, v = -7)$ c) $(u = -2, v = -7)$ d) $(0, -1)$
- The gradient of the curve $y + px + qy = 0$ at $(1, 1)$ is $1/2$. The values of p and q are
 a) $(-1, 1)$ b) $(2, -1)$ c) $(1, 2)$ d) $(0, -1)$
- If $xy = 1$ then $y^2 + dy/dx$ is equal to
 a) 1 b) 0 c) -1 d) none of these
- The derivative of the function $\sqrt{x+\sqrt{x}}$ is
 a) $\frac{1}{2\sqrt{x+\sqrt{x}}}$ b) $1 + \frac{1}{2\sqrt{x}}$ c) $\frac{1}{2(x+\sqrt{x})} \left(1 + \frac{1}{2\sqrt{x}}\right)$ d) none of these



12. Given $e^{xy} - 4xy = 0$, $\frac{dy}{dx}$ can be proved to be
a) $-y/x$ b) y/x c) x/y d) none of these
13. If $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$, $\frac{dy}{dx}$ can be expressed as
a) $\frac{x}{a}$ b) $\frac{x}{\sqrt{x^2 - a^2}}$ c) $\frac{1}{\sqrt{\frac{x^2}{a^2} - 1}}$ d) none of these
14. If $\log(x/y) = x + y$, $\frac{dy}{dx}$ may be found to be
a) $\frac{y(1-x)}{x(1+y)}$ b) $\frac{y}{x}$ c) $\frac{1-x}{1+y}$ d) none of these
15. If $f(x, y) = x^3 + y^3 - 3axy = 0$, $\frac{dy}{dx}$ can be found out as
a) $\frac{ay - x^2}{y^2 + ax}$ b) $\frac{ay - x^2}{y^2 - ax}$ c) $\frac{ay + x^2}{y^2 + ax}$ d) none of these
16. Given $x = at^2$, $y = 2at$; $\frac{dy}{dx}$ is calculated as
a) t b) $-1/t$ c) $1/t$ d) none of these
17. Given $x = 2t + 5$, $y = t^2 - 2$; $\frac{dy}{dx}$ is calculated as
a) t b) $-1/t$ c) $1/t$ d) none of these
18. If $y = \frac{1}{\sqrt{x}}$ then $\frac{dy}{dx}$ is equal to
a) $\frac{1}{2x\sqrt{x}}$ b) $\frac{-1}{x\sqrt{x}}$ c) $-\frac{1}{2x\sqrt{x}}$ d) none of these
19. If $x = 3t^2 - 1$, $y = t^3 - t$, then $\frac{dy}{dx}$ is equal to
a) $\frac{3t^2 - 1}{6t}$ b) $3t^2 - 1$ c) $\frac{3t - 1}{6t}$ d) none of these



20. The slope of the tangent to the curve $y = \sqrt{4-x^2}$ at the point, where the ordinate and the abscissa are equal, is
 a) -1 b) 1 c) 0 d) none of these
21. The slope of the tangent to the curve $y = x^2 - x$ at the point, where the line $y = 2$ cuts the curve in the 1st quadrant, is
 a) 2 b) 3 c) -3 d) none of these
22. For the curve $x^2 + y^2 + 2gx + 2hy = 0$, the value of $\frac{dy}{dx}$ at $(0, 0)$ is
 a) -g/h b) g/h c) h/g d) none of these
23. If $y = \frac{e^{3x} - e^{2x}}{e^{3x} + e^{2x}}$, then $\frac{dy}{dx}$ is equal to
 a) $2e^{5x}$ b) $1/(e^{5x} + e^{2x})^2$ c) $e^{5x}/(e^{5x} + e^{2x})$ d) none of these
24. If $x^y \cdot y^x = M$, M is constant then $\frac{dy}{dx}$ is equal to
 a) $-\frac{y}{x}$ b) $\frac{-y(y+x \log y)}{x(y \log x + x)}$ c) $\frac{y+x \log y}{y \log x + x}$ d) none of these
25. Given $x = t + t^{-1}$ and $y = t - t^{-1}$ the value of $\frac{dy}{dx}$ at $t = 2$ is
 a) 3/5 b) -3/5 c) 5/3 d) none of these
26. If $x^3 - 2x^2 y^2 + 5x + y - 5 = 0$ then $\frac{dy}{dx}$ at $x = 1, y = 1$ is equal to
 a) 4/3 b) -4/3 c) 3/4 d) none of these
27. The derivative of $x^2 \log x$ is
 a) $1+2\log x$ b) $x(1 + 2 \log x)$ c) $2 \log x$ d) none of these
28. The derivative of $\frac{3-5x}{3+5x}$ is
 a) $30/(3 + 5x)^2$ b) $1/(3 + 5x)^2$ c) $-30/(3 + 5x)^2$ d) none of these
29. Let $y = \sqrt{2x} + 3^{2x}$ then $\frac{dy}{dx}$ is equal to
 a) $(1/\sqrt{2x}) + 2.3^{2x} \log_e 3$ b) $1/\sqrt{2x}$
 c) $2.3^{2x} \log_e 3$ d) none of these



30. The derivative of $\log e^x \left\{ \frac{(x-2)}{x+2} \right\}^{3/4}$ is
- a) $\frac{x^2+1}{x^2+4}$ b) $\frac{x^2-1}{x^2-4}$ c) $\frac{1}{x^2-4}$ d) none of these
31. The derivative of e^{3x^2-6x+2} is
- a) $30(1-5x)^5$ b) $(1-5x)^5$ c) $6(x-1)e^{3x^2-6x+2}$ d) none of these
32. If $y = \frac{e^x+1}{e^x-1}$ then $\frac{dy}{dx}$ is equal to
- a) $\frac{-2e^x}{(e^x-1)^2}$ b) $\frac{-2e^x}{(e^x-1)^2}$ c) $\frac{-2}{(e^x-1)^2}$ d) none of these
33. If $f(x) = \left\{ \frac{(a+x)}{(1+x)} \right\}^{a+1+2x}$ the value of $f'(0)$ is
- a) a^{a+1} b) $a^{a+1} \left\{ \frac{(1-a^2)}{a+2\log a} \right\}$ c) $2 \log a$ d) none of these
34. If $x = at^2$ $y = 2at$ then $\left[\frac{dy}{dx} \right]_{t=2}$ is equal to
- a) $1/2$ b) -2 c) $-1/2$ d) none of these
35. Let $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$ then $f'(2)$ is equal to
- a) $3/4$ b) $1/2$ c) 0 d) none of these
36. If $f(x) = x^2 - 6x + 8$ then $f'(5) - f'(8)$ is equal to
- a) $f'(2)$ b) $3f'(2)$ c) $2f'(2)$ d) none of these
37. If $y = \left(x + \sqrt{x^2+m^2} \right)^n$ then dy/dx is equal to
- a) ny b) $ny / \sqrt{x^2+m^2}$ c) $-ny / \sqrt{x^2+m^2}$ d) none of these
38. If $y = + \sqrt{x/m} + \sqrt{m/x}$ then $2xy \frac{dy}{dx} - x/m + m/x$ is equal to
- a) 0 b) 1 c) -1 d) none of these
39. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n}$ then $\frac{dy}{dx} - y$ is proved to be
- a) 1 b) -1 c) 0 d) none of these

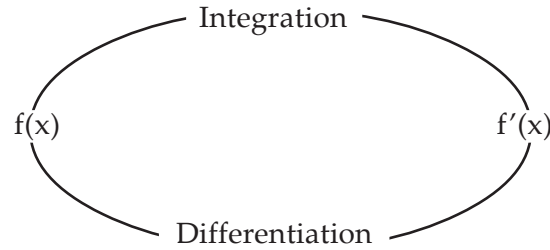


40. If $f(x) = x^k$ and $f'(1) = 10$ the value of k is
 a) 10 b) -10 c) 1/10 d) none of these
41. If $y = \sqrt{x^2+m^2}$ then y_{y_1} (where $y_1 = dy/dx$) is equal to
 a) $-x$ b) x c) $1/x$ d) none of these
42. If $y = e^x + e^{-x}$ then $\frac{dy}{dx} - \sqrt{y^2 - 4}$ is equal to
 a) 1 b) -1 c) 0 d) none of these
43. The derivative of $(x^2-1)/x$ is
 a) $1 + 1/x^2$ b) $1 - 1/x^2$ c) $1/x^2$ d) none of these
44. The differential coefficients of $(x^2 + 1)/x$ is
 a) $1 + 1/x^2$ b) $1 - 1/x^2$ c) $1/x^2$ d) none of these
45. If $y = e^{\sqrt{2x}}$ then $\frac{dy}{dx}$ is equal to _____
 a) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ b) $e^{\sqrt{2x}}$ c) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ d) none of these
46. If $y = \sqrt{x}^{\sqrt{x}}$ then $\frac{dy}{dx}$ is equal to _____
 a) $\frac{y^2}{2 - y \log x}$ b) $\frac{y^2}{x(2 - y \log x)}$ c) $\frac{y^2}{\log x}$ d) none of these
47. If $x = (1 - t^2)/(1 + t^2)$ $y = 2t/(1 + t^2)$ then dy/dx at $t = 1$ is _____
 a) 1/2 b) 1 c) 0 d) none of these
48. $f(x) = x^2/e^x$ then $f'(1)$ is equal to _____
 a) $-1/e$ b) $1/e$ c) e d) none of these
49. If $y = (x + \sqrt{x^2-1})^m$ then $(x^2-1)(dy/dx)^2 - m^2y^2$ is proved to be
 a) -1 b) 1 c) 0 d) none of these
50. If $f(x) = \frac{4 - 2x}{2 + 3x + 3x^2}$ then the values of x for which $f'(x) = 0$ is
 a) $2(1 \pm \sqrt{\frac{5}{3}})$ b) $(1 \pm \sqrt{3})$ c) 2 d) none of these

(B) INTEGRAL CALCULUS

9.B.1 INTEGRATION

Integration is the reverse process of differentiation.



we know

$$\frac{d}{dx} \left(\frac{x^{n+1}}{n+1} \right) = \frac{(n+1)x^n}{(n+1)}$$

o or $\frac{d}{dx} \left(\frac{x^{n+1}}{n+1} \right) = x^n$ (1)

Integration is the inverse operation of differentiation and is denoted by the symbol \int .

Hence, from equation (1), it follows that

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

i.e. Integral of x^n with respect to variable x is equal to $\frac{x^{n+1}}{n+1}$

Thus if we differentiate $\frac{(x^{n+1})}{n+1}$ we can get back x^n

Again if we differentiate $\frac{(x^{n+1})}{n+1} + c$ and c being a constant, we get back the same x^n .

i.e. $\frac{d}{dx} \left[\frac{x^{n+1}}{n+1} + c \right] = x^n$

Hence $\int x^n dx = \frac{(x^{n+1})}{n+1} + c$ and this c is called the constant of integration.

Integral calculus was primarily invented to determine the area bounded by the curves dividing the entire area into infinite number of infinitesimal small areas and taking the sum of all these small areas.

9.B.2 BASIC FORMULAS

i) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$ (If $n=-1, \frac{x^{n+1}}{n+1} = \frac{1}{0}$ is not defined)

ii) $\int dx = x$, since $\int 1 dx = \int x^0 dx = \frac{x^1}{1} = x$



iii) $\int e^x dx = e^x + c$, since $\frac{d}{dx} e^x = e^x$

iv) $\int e^{ax} dx = \frac{e^{ax}}{a} + c$, since $\frac{d}{dx} \left(\frac{e^{ax}}{a} \right) = e^{ax}$

v) $\int \frac{dx}{x} = \log x + c$, since $\frac{d}{dx} \log x = \frac{1}{x}$

vi) $\int a^x dx = \frac{a^x}{\log_e a} + c$, since $\frac{d}{dx} \left(\frac{a^x}{\log_e a} \right) = a^x$

Note: In the answer for all integral sums we add +c (constant of integration) since the differentiation of constant is always zero.

Elementary Rules:

$\int c f(x) dx = c \int f(x) dx$ where c is constant.

$\int \{ f(x) dx \pm g(x) \} dx = \int f(x) dx \pm \int g(x) dx$

Examples : Find (a) $\int \sqrt{x} dx$, (b) $\int \frac{1}{\sqrt{x}} dx$, (c) $\int e^{-3x} dx$ (d) $\int 3^x dx$ (e) $\int x\sqrt{x} dx$.

Solution: (a) $\int \sqrt{x} dx = x^{1/2+1} / (1/2+1) = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3} + c$

(b) $\int \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx = \frac{x^{-1/2+1}}{-1/2+1} + c = 2\sqrt{x} + c$ where c is arbitrary constant.

(c) $\int e^{-3x} dx = \frac{e^{-3x}}{-3} + c = -\frac{1}{3} e^{-3x} + c$

(d) $\int 3^x dx = \frac{3^x}{\log_e 3} + c$.

(e) $\int x\sqrt{x} dx = \int x^{3/2} dx = \frac{x^{3/2+1}}{3/2+1} = \frac{2}{5} x^{5/2} + c$.

Examples : Evaluate the following integral:

i) $\int (x + 1/x)^2 dx = \int x^2 dx + 2 \int dx + \int dx / x^2$



$$= \frac{x^3}{3} + 2x + \frac{x^{-2+1}}{-2+1}$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + c$$

$$\text{ii) } \int \sqrt{x} (x^3 + 2x - 3) dx = \int x^{7/2} dx + 2 \int x^{3/2} dx - 3 \int x^{1/2} dx$$

$$= \frac{x^{7/2+1}}{7/2+1} + \frac{2x^{3/2+1}}{3/2+1} - \frac{3x^{1/2+1}}{1/2+1}$$

$$= \frac{2x^{9/2}}{9} + \frac{4x^{5/2}}{5} - 2x^{3/2} + c$$

$$\text{iii) } \int e^{3x} + e^{-3x} dx = \int e^{2x} dx + \int e^{-4x} dx$$

$$= \frac{e^{2x}}{2} + \frac{e^{-4x}}{-4} = \frac{e^{2x}}{2} - \frac{1}{4e^{4x}} + c$$

$$\text{iv) } \int \frac{x^2}{x+1} dx = \int \frac{x^2 - 1 + 1}{x+1} dx$$

$$= \int \frac{(x^2 - 1)}{x+1} dx + \int \frac{dx}{x+1}$$

$$= \int (x-1) dx + \log(x+1) = \frac{x^2}{2} - x + \log(x+1) + c$$

$$\text{v) } \int \frac{x^3 + 5x^2 - 3}{(x+2)} dx$$

$$\text{By simple division } \int \frac{x^3 + 5x^2 - 3}{(x+2)} dx$$

$$= \int \left\{ x^2 + 3x - 6 + \frac{9}{(x-2)} \right\} dx$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} - 6x + 9 \log(x+2) + c$$

9.B.3 METHOD OF SUBSTITUTION (CHANGE OF VARIABLE)

It is sometime possible by a change of independent variable to transform a function into another which can be readily integrated.

We can show the following rules.



To put $z = f(x)$ and also to adjoin $dz = f'(x) dx$

Example: $\int F\{h(x)\} h'(x) dx$, take $e^z = h(x)$ and to adjust $dz = h'(x) dx$

then integrate $\int F(z) dz$ using normal rule.

Example: $\int (2x+3)^7 dx$

We put $(2x+3) = t \Rightarrow$ so $2 dx = dt$ or $dx = dt / 2$

$$\text{Therefore } \int (2x+3)^7 dx = \frac{1}{2} \int t^7 dt = \frac{t^8}{2 \times 8} = \frac{t^8}{16} = \frac{(2x+3)^8}{16} + c$$

This method is known as Method of Substitution

Example: $\int \frac{x^3}{(x^2+1)^3} dx$ We put $(x^2+1) = t$

so $2x dx = dt$ or $x dx = dt / 2$

$$= \int \frac{x^2 \cdot x}{t^3} dx$$

$$= \frac{1}{2} \int \frac{t-1}{t^3} dt$$

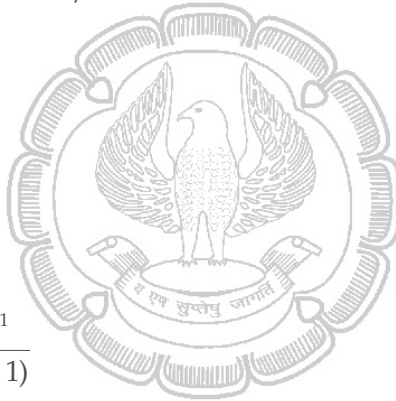
$$= \frac{1}{2} \int \frac{dt}{t^2} - \frac{1}{2} \int \frac{dt}{t^3}$$

$$= \frac{1}{2} \times \frac{t^{-2+1}}{(-2+1)} - \frac{1}{2} \times \frac{t^{-3+1}}{(-3+1)}$$

$$= -\frac{1}{2} \cdot \frac{1}{t} + \frac{1}{4} \cdot \frac{1}{t^2}$$

$$= \frac{1}{4} \cdot \frac{1}{t^2} - \frac{1}{2} \cdot \frac{1}{t}$$

$$= \frac{1}{4} \cdot \frac{1}{(x^2+1)} - \frac{1}{2} \cdot \frac{1}{(x^2+1)} + c$$



IMPORTANT STANDARD FORMULAS

aa) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a}$

b) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x}$

c) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right|$



$$d) \int \frac{dx}{\sqrt{x^2 - a^2}} = \log(x + \sqrt{x^2 - a^2})$$

$$e) \int e^x \{f(x) + f'(x)\} dx = e^x f(x)$$

$$f) \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2})$$

$$g) \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2})$$

$$h) \int \frac{f'(x)}{f(x)} dx = \log f(x)$$

Examples : (a) $\int \frac{e^x}{e^{2x} - 4} dx = \int \frac{dz}{z^2 - 2^2}$ where $z = e^x$ $dz = e^x dx$

$$= \frac{1}{4} \log \left(\frac{e^x - 2}{e^x + 2} \right) + c$$

(b) $\int \frac{1}{x + \sqrt{x^2 - 1}} dx = \int \frac{x - \sqrt{x^2 - 1}}{(x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1})} dx = \int (x - \sqrt{x^2 - 1}) dx$

$$= \frac{x^2}{2} - \frac{x}{2} \sqrt{x^2 - 1} + \frac{1}{2} \log(x + \sqrt{x^2 - 1}) + c$$

(c) $\int e^x (x^3 + 3x^2) dx = \int e^x \{f(x) + f'(x)\} dx$, where $f(x) = x^3$
 [by (e) above)] = $e^x x^3 + c$

9.B.4 INTEGRATION BY PARTS

$$\int u v dx = u \int v dx - \int \left[\frac{d(u)}{dx} \int v dx \right] dx$$

where u and v are two different functions of x

Evaluate:

i) $\int x e^x dx$

Integrating by parts we see

$$\int x e^x dx = x \int e^x dx - \int \left\{ \frac{d}{dx}(x) \int e^x dx \right\} dx$$

$$= x e^x - \int 1 \cdot e^x dx = x e^x - e^x + c$$



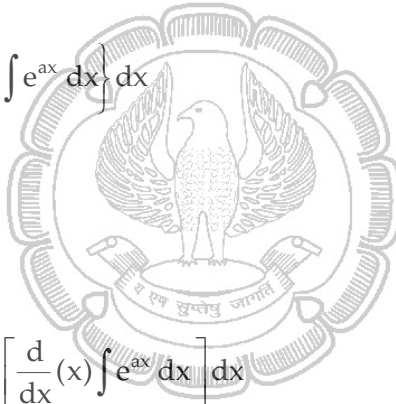
ii) $\int x \log x \, dx$

Integrating by parts,

$$\begin{aligned} & \log x \int x \, dx - \int \left\{ \frac{d}{dx} (\log x) \int x \, dx \right\} dx \\ &= \frac{x^2}{2} \log x - \int \left[\frac{1}{x} \cdot \frac{x^2}{2} \right] dx \\ &= \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx \\ &= \frac{x^2}{2} \log x - \frac{x^2}{4} + c \text{ Ans.} \end{aligned}$$

iii) $\int x^2 e^{ax} \, dx$

$$\begin{aligned} &= x^2 \int e^{ax} \, dx - \int \left\{ \frac{d}{dx} (x^2) \int e^{ax} \, dx \right\} dx \\ &= \frac{x^2}{a} e^{ax} - \int 2x \cdot \frac{e^{ax}}{a} \, dx \\ &= \frac{x^2}{a} e^{ax} - \frac{2}{a} \int x \cdot e^{ax} \, dx \\ &= \frac{x^2}{a} e^{ax} - \frac{2}{a} \times \int e^{ax} \, dx - \int \left[\frac{d}{dx} (x) \int e^{ax} \, dx \right] dx \\ &= \frac{x^2 e^{ax}}{a} - \frac{2}{a} \left[\frac{x e^{ax}}{a} - \int 1 \cdot \frac{e^{ax}}{a} \, dx \right] \\ &= \frac{x^2 e^{ax}}{a} - \frac{2x e^{ax}}{a^2} + \frac{2}{a^3} e^{ax} + c \end{aligned}$$



9. B.5 METHOD OF PARTIAL FRACTION

Type I :

Example : $\int \frac{(3x + 2) \, dx}{(x - 2)(x - 3)}$

Solution : let $\frac{(3x + 2)}{(x - 2)(x - 3)}$

$$= \frac{A}{(x - 2)} + \frac{B}{(x - 3)}$$



[Here degree of the numerator must be lower than that of the denominator; the denominator contains non-repeated linear factor]

$$\text{so } 3x + 2 = A(x - 3) + B(x - 2)$$

We put $x = 2$ and get

$$3 \cdot 2 + 2 = A(2 - 3) + B(2 - 2) \quad \Rightarrow A = -8$$

we put $x = 3$ and get

$$3 \cdot 3 + 2 = A(3 - 3) + B(3 - 2) \quad \Rightarrow B = 11$$

$$\int \frac{(3x+2)}{(x-2)(x-3)} dx = -8 \int \frac{dx}{x-2} + 11 \int \frac{dx}{x-3}$$

$$= -\log(x-2) + 11 \log(x-3) + c$$

Type II:

Example : $\int \frac{(3x+2) dx}{(x-2)^2 (x-3)}$

Solution : let $\frac{(3x+2)}{(x-2)^2 (x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)}$

$$\text{or } 3x + 2 = A(x-2)(x-3) + B(x-3) + C(x-2)^2$$

Comparing coefficients of x^2 , x and the constant terms of both sides, we find

$$A + C = 0 \dots\dots\dots(i)$$

$$-5A + B - 4C = 3 \dots\dots(ii)$$

$$6A - 3B + 4C = 2 \dots\dots(iii)$$

$$\text{By (ii) + (iii) } A - 2B = 5 \dots\dots(iv)$$

$$(i) - (iv) \quad 2B + C = -5 \dots\dots(v)$$

$$\text{From (iv) } A = 5 + 2B$$

$$\text{From (v) } C = -5 - 2B$$

$$\text{From (ii) } -5(5 + 2B) + B - 4(-5 - 2B) = 3$$

$$\text{or } -25 - 10B + B + 20 + 8B = 3$$

$$\text{or } -B - 5 = 3$$

$$\text{or } B = -8 \quad A = 5 - 16 = -11, \text{ from (iv) } C = -A = 11$$

Therefore $\int \frac{(3x+2) dx}{(x-2)^2 (x-3)}$

$$= -11 \int \frac{dx}{(x-2)} - 8 \int \frac{dx}{(x-2)^2} + 11 \int \frac{dx}{(x-3)}$$



$$= -11 \log (x-2) + \frac{8}{(x-2)} + 11 \log (x-3)$$

$$= 11 \log \frac{(x-3)}{(x-2)} + \frac{8}{(x-2)} + c \text{ Ans.}$$

Type III:

Example: $\int \frac{(3x^2 - 2x + 5)}{(x-1)^2 (x^2 + 5)} dx$

Solution: Let $\frac{3x^2 - 2x + 5}{(x-1)^2 (x^2 + 5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+5}$

so $3x^2 - 2x + 5 = A(x^2 + 5) + (Bx + C)(x-1)$

Equating the coefficients of x^2 , x and the constant terms from both sides we get

$A + B = 3$ (i)

$C - B = -2$ (ii)

$5A - C = 5$ (iii)

by (i) + (ii) $A + C = 1$ (iv)

by (iii) + (iv) $6A = 6$ (v)

or $A = 1$

therefore $B = 3 - 1 = 2$ and $C = 0$

Thus $\int \frac{(3x^2 - 2x + 5)}{(x-1)^2 (x^2 + 5)} dx$

$$= \int \frac{dx}{x-1} + \int \frac{2x}{x^2+5} dx = \log (x-1) + \log (x^2 + 5)$$

$$= \log (x^2 + 5) (x-1) + c \text{ Ans.}$$

Example: $\int \frac{dx}{x(x^3 + 1)}$

Solution : $\int \frac{dx}{x(x^3 + 1)}$

$$= \int \frac{x^2 dx}{x^3 (x^3 + 1)} \quad \text{we put } x^3 = z, 3x^2 dx = dz$$



$$\begin{aligned} &= \frac{1}{3} \int \frac{dz}{z(z+1)} \\ &= \frac{1}{3} \int \left(\frac{1}{z} - \frac{1}{z+1} \right) dz \\ &= \frac{1}{3} [\log z - \log(z+1)] \\ &= \frac{1}{3} \log \left(\frac{x^3}{x^3+1} \right) + c \end{aligned}$$

Example : Find the equation of the curve where slope at (x, y) is $9x$ which passes through the origin.

Solution : $\frac{dy}{dx} = 9x$

$$\therefore \int dy = \int 9x \, dx \quad \text{or } y = 9x^2 / 2 + c$$

Since it passes through the origin, $c = 0$; thus required eqn. is $9x^2 = 2y$.

9.B.6 DEFINITE INTEGRATION

Suppose $F(x) \, dx = f(x)$

As x changes from a to b the value of the integral changes from $f(a)$ to $f(b)$. This is as

$$\int_a^b F(x) \, dx = f(b) - f(a)$$

' b ' is called the upper limit and ' a ' the lower limit of integration. We shall first deal with indefinite integral and then take up definite integral.

Example : $\int_0^2 x^5 \, dx$

Solution : $\int_0^2 x^5 \, dx = \frac{x^6}{6}$

$$\begin{aligned} \int_0^2 x^5 \, dx &= \left(\frac{x^6}{6} \right)_0^2 \\ &= \frac{1}{6} (2^6 - 0) = 64/6 = 32/3 = 10.666 \end{aligned}$$



Note: In definite integration the constant **C** should not be added

Example: $\int_1^2 (x^2 - 5x + 2)dx$

Solution: $\int_1^2 (x^2 - 5x + 2)dx = \frac{x^3}{3} - \frac{5x^2}{2} + 2x$ Now, $\int_1^2 (x^2 - 5x + 2)dx = \left[\frac{x^3}{3} - \frac{5x^2}{2} + 2x \right]_1^2$
 $= \left[\frac{2^3}{3} - \frac{5 \times 2^2}{2} + 2 \times 2 \right] - \left[\frac{1}{3} - \frac{5}{2} + 2 \right] = -19/6$

9. B.7 IMPORTANT PROPERTIES

Important Properties of Definite Integral

(I) $\int_a^b f(x)dx = \int_a^b f(t) dt$ (II) $\int_a^b f(x) dx = - \int_b^a f(x) dx$

(III) $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx, a < c < b$

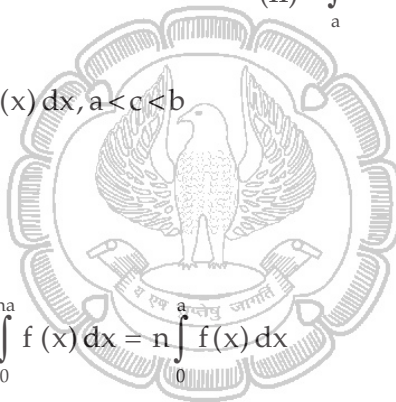
(IV) $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

(V) When $f(x) = f(a+x) = \int_0^{na} f(x) dx = n \int_0^a f(x) dx$

(VI) $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ if $f(-x) = f(x)$
 $= 0$ if $f(-x) = -f(x)$

Example : $\int_0^2 \frac{x^2 dx}{x^2 + (2-x)^2}$

Solution : Let $I = \int_0^2 \frac{x^2 dx}{x^2 + (2-x)^2}$
 $= \int_0^2 \frac{(2-x)^2 dx}{(2-x)^2 + x^2}$ [by prop IV]





$$\begin{aligned} \therefore 2I &= \int_0^2 \frac{x^2 dx}{x^2 + (2-x)^2} + \int_0^2 \frac{(2-x)^2}{(2-x)^2 + x^2} \\ &= \int_0^2 \frac{x^2 + (2-x)^2}{x^2 + (2-x)^2} dx \\ &= \int_0^2 dx = [x]_0^2 = 2-0 = 2 \end{aligned}$$

or $I = 2/2 = 1$

Example : Evaluate $\int_{-2}^2 \frac{x^4 dx}{a^{10} - x^{10}}$ ($a > 2$)

Solution : $\frac{x^4 dx}{a^{10} - x^{10}} = \frac{x^4 dx}{(a^5)^2 - (x^5)^2}$

let $x^5 = t$ so that $5x^4 dx = dt$

Now $\int \frac{x^4 dx}{(a^5)^2 - (x^5)^2}$

$= \frac{1}{5} \int \frac{5x^4 dx}{(a^5)^2 - (x^5)^2}$

$= \frac{1}{5} \int \frac{dt}{(a^5)^2 - t^2}$

$= \frac{1}{10a^5} \log \frac{a^5 + x^5}{a^5 - x^5}$ (by standard formula b)

Therefore, $\int_{-2}^2 \frac{x^4 dx}{a^{10} - x^{10}}$

$= 2 \int_0^2 \frac{x^4 dx}{a^{10} - x^{10}}$ (by prop. VI)

$= 2 \times \frac{1}{10a^5} \log \left[\frac{a^5 + x^5}{a^5 - x^5} \right]_0^2$

$= \frac{1}{5a^5} \log \frac{a^5 + 32}{a^5 - 32}$ Ans.



EXERCISE 9 (B) [K = constant]

Choose the most appropriate option (a) (b) (c) or (d)

- Evaluate $\int 5x \, dx$ and the answer will be
 (a) $5 / 3x^3 + k$ (b) $\frac{5x^3}{3} + k$ (c) $5x^3$ (d) none of these
- Integration of $3 - 2x - x^4$ will become
 (a) $-x^2 - x^5 / 5$ (b) $3x - x^2 - \frac{x^5}{5} + k$ (c) $3x - x^2 + \frac{x^5}{5} + k$ (d) none of these
- Given $f(x) = 4x^3 + 3x^2 - 2x + 5$ and $\int f(x) \, dx$ is
 (a) $x^4 + x^3 - x^2 + 5x$ (b) $x^4 + x^3 - x^2 + 5x + k$
 (c) $12x^2 + 6x - 2x^2$ (d) none of these
- Evaluate result of $\int (x^2 - 1) \, dx$ is
 (a) $x^5/5 - 2/3 x^3 + x + k$ (b) $\frac{x^3}{3} - x + k$
 (c) $2x$ (d) none of these
- $\int (1 - 3x)(1 + x) \, dx$ is equal to
 (a) $x - x^2 - x^3$ (b) $x^3 - x^2 + x$ (c) $x - x^2 - x^3 + k$ (d) none of these
- $\int [\sqrt{x} - 1/\sqrt{x}] \, dx$ is equal to
 (a) $\frac{2}{3}x^{3/2} - 2x^{1/2} + k$ (b) $\frac{2}{3}\sqrt{x} - 2\sqrt{x} + k$ (c) $\frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}} + k$ (d) none of these
- The integral of $px^3 + qx^2 + rx + w/x$ is equal to
 (a) $px^2 + qx + r + k$ (b) $px^3/3 + qx^2/2 + rx$
 (c) $3px + 2q - w/x^2$ (d) none of these
- Use method of substitution to integrate the function $f(x) = (4x + 5)^6$ and the answer is
 (a) $1/28 (4x + 5)^7 + k$ (b) $(4x + 5)^7/7 + k$ (c) $(4x + 5)^7/7$ (d) none of these
- Use method of substitution to evaluate $\int x(x^2 + 4)^5 \, dx$ and the answer is
 (a) $(x^2 + 4)^6 + k$ (b) $1/12 (x^2 + 4)^6 + k$
 (c) $(x^2 + 4)^6 / + k$ (d) none of these
- Integrate $(x + a)^n$ and the result will be
 (a) $\frac{(x + a)^{n+1}}{n + 1} + k$ (b) $\frac{(x + a)^{n+1}}{n + 1}$
 (c) $(x + a)^{n+1}$ (d) none of these



11. $\int 8x^2 / (x^3 + 2)^3 dx$ is equal to
- (a) $-4/3(x^3 + 2)^2$ (b) $-\frac{4}{3(x^3 + 2)^2} + k$
(c) $\frac{4}{3(x^3 + 2)^2} + k$ (d) none of these
12. Using method of partial fraction find the integration of $f(x)$ when $f(x) = 1/x^2 - a^2$ and the answer is
- (a) $\log x - \frac{a}{x+a} + k$ (b) $\log(x - a) - \log(x + a)$
(c) $\frac{1}{2a \log} - \frac{a}{x+a} + k$ (d) none of these
13. Use integration by parts to evaluate $\int x^2 e^{3x} dx$ and the answer is
- (a) $x^2 e^{3x}/3 - 2x e^{3x}/9 + 2/27 e^{3x} + k$ (b) $x^2 e^{3x} - 2x e^{3x} + 2e^{3x} + k$
(c) $e^{3x}/3 - x e^{3x}/9 + 2e^{3x} + k$ (d) none of these
14. $\int \log x dx$ is equal to
- (a) $x \log x$ (b) $x \log x - x^2 + k$ (c) $x \log x + k$ (d) none of these
15. $\int x e^x dx$ is
- (a) $(x - 1)e^x + k$ (b) $(x - 1) e^x$ (c) $x e^x + k$ (d) none of these
16. $\int (\log x)^2 dx$ and the result is
- (a) $x (\log x)^2 - 2x \log x + 2x$ (b) $x (\log x)^2 - 2x$
(c) $2x \log x - 2x$ (d) none of these
17. Using method of partial fraction to evaluate $\int (x + 5) dx / (x + 1)(x + 2)^2$ and the final answer becomes
- (a) $4 \log(x + 1) - 4 \log(x + 2) + 3/x + 2 + k$
(b) $4 \log(x + 2) - 3/x + 2 + k$
(c) $4 \log(x + 1) - 4 \log(x + 2)$
(d) none of these
18. Evaluate $\int_0^1 (2x^2 - x^3) dx$ and the value is
- (a) $4/3 + k$ (b) $5/12$ (c) $-4/3$ (d) none of these



19. Evaluate $\int_2^4 (3x-2)^2 dx$ and the value is
 (a) 104 (b) 100 (c) 10 (d) none of these.
20. Evaluate $\int_0^1 xe^x dx$ and the value is
 (a) -1 (b) 10 (c) 10/9 (d) +1
21. $\int x^x (1 + \log x) dx$ is equal to
 (a) $x^x \log x + k$ (b) $e^{x^2} + k$ (c) $\frac{x^2}{2} + k$ (d) $x^x + c$
22. If $f(x) = \sqrt{1+x^2}$ then $\int f(x)dx$ is
 (a) $\frac{2}{3} x (1+x^2)^{3/2} + k$ (b) $\frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log(x+\sqrt{x^2+1})$
 (c) $\frac{2}{3} x (1+x^2)^{3/2} + k$ (d) none of these
23. $\int d(x^2+1)/\sqrt{x^2+2}$ is equal to
 (a) $2\sqrt{x^2+2} + k$ (b) $\sqrt{x^2+2} + k$ (c) $1/(x^2+2)^{3/2} + k$ (d) none of these
24. $\int (e^x + e^{-x})^2 (e^x - e^{-x}) dx$ is
 (a) $\frac{1}{3}(e^x - e^{-x})^3 + k$ (b) $\frac{1}{2}(e^x - e^{-x})^2 + k$
 (c) $e^x + k$ (d) none of these
25. $\int_0^a [f(x) + f(-x)] dx$ is equal to
 (a) $\int_0^a 2f(x) dx$ (b) $\int_{-a}^a f(x) dx$ (c) 0 (d) $\int_{-a}^a -f(-x) dx$
26. $\int xe^x/(x+1)^2 dx$ is equal to
 (a) $e^x/(x+1) + k$ (b) $e^x/x + k$ (c) $e^x + k$ (d) none of these
27. $\int (x^4 + 3/x) dx$ is equal to
 (a) $x^5/5 + 3 \log |x|$ (b) $1/5 x^5 + 3 \log |x| + k$
 (c) $1/5 x^5 + k$ (d) none of these



28. Evaluate the integral $\int (1-x)^3/x \, dx$ and the answer is equal to
(a) $\log|x| - 3x + 3/2x^2 + k$ (b) $\log x - 2 + 3x^2 + k$
(c) $\log x + 3x^2 + k$ (d) none of these
29. The equation of the curve in the form $y = f(x)$ if the curve passes through the point $(1, 0)$ and $f'(x) = 2x - 1$ is
(a) $y = x^2 - x$ (b) $x = y^2 - y$ (c) $y = x^2$ (d) none of these
30. Evaluate $\int_1^4 (2x+5) \, dx$ and the value is
(a) 3 (b) 10 (c) 30 (d) none of these
31. $\int_1^2 \frac{2x}{1+x^2} \, dx$ is equal to
(a) $\log_e(5/2)$ (b) $\log_e 5 - \log_e 2 + k$
(c) $\log_e(2/5)$ (d) none of these
32. $\int_0^4 \sqrt{3x+4} \, dx$ is equal to
(a) $9/112$ (b) $112/9$ (c) $11/9$ (d) none of these
33. $\int_0^2 \frac{x+2}{x+1} \, dx$ is
(a) $2 + \log_e 2$ (b) $2 + \log_e 3$ (c) $\log_e 3$ (d) none of these
34. Evaluate $\int_1^{e^2} \frac{dx}{x(1+\log x^2)}$ and the value is
(a) $3/2$ (b) $1/3$ (c) $26/3$ (d) $1/2 (\log_e 5)$
35. $\int_0^4 \frac{(x+1)(x+4)}{\sqrt{x}} \, dx$ is equal to
(a) $51\frac{1}{5}$ (b) $48/5$ (c) 48 (d) $55\frac{7}{15}$
36. The equation of the curve which passes through the point $(1, 3)$ and has the slope $4x - 3$ at any point (x, y) is
(a) $y = 2x^3 - 3x + 4$ (b) $y = 2x^2 - 3x + 4$
(c) $x = 2y^2 - 3y + 4$ (d) none of these



37. The value of $\int_2^3 f(5-x) dx - \int_2^3 f(x) dx$ is
 (a) 1 (b) 0 (c) -1 (d) none of these
38. $\int (x+1)e^x/x^2 dx$ is equal to
 (a) $e^x/x + k$ (b) $e^{-x}/x + k$ (c) $-e^x/x + k$ (d) none of these
39. $\int \frac{e^x(x \log x + 1)}{x} dx$ is equal to
 (a) $e^x \log x + k$ (b) $e^x + k$ (c) $\log x + k$ (d) none of these
40. $\int \log x^2 dx$ is equal to
 (a) $x(\log x - 1) + k$ (b) $2x(\log x - 1) + k$
 (c) $2(\log x - 1) + k$ (d) none of these
41. $\int_1^2 x \log x dx$ is equal to
 (a) $2 \log 2$ (b) $-3/4$ (c) $2 \log 2 - 3/4$ (d) none of these
42. Integrate $(x^2 - 1)/x^2 e^{x+1/x}$ and hence evaluate $\int_1^2 (x^2 - 1)/x^2 e^{x+1/x} dx$ and the value is
 (a) $e^2(\sqrt{e} - 1)$ (b) $e^2[\sqrt{e} - 1] + k$ (c) $e^2\sqrt{e}$ (d) none of these
43. $\int_0^2 3x^2 dx$ is
 (a) 7 (b) -8 (c) 8 (d) none of these
44. Evaluate $\int \frac{(2-x)e^x}{(1-x)^2} dx$ and the value is
 (a) $\frac{e^x}{1-x} + k$ (b) $e^x + k$ (c) $\frac{1}{1-x} + k$ (d) none of these
45. Using integration by parts integrate $x^3 \log x$ and the integral is
 (a) $x^4/16 + k$ (b) $x^4/16 (4 \log x - 1) + k$
 (c) $4 \log x - 1 + k$ (d) none of these
46. $\int \log(\log x)/x dx$ is
 (a) $\log(\log x - 1) + k$ (b) $\log x - 1 + k$
 (c) $[\log(\log x - 1)] \log x + k$ (d) none of these



47. $\int (\log x)^2 \times dx$ is equal to

- (a) $\frac{x^2}{2} \left[(\log x)^2 - \log x + \frac{1}{2} \right] + k$ (b) $(\log x)^2 - \log x + \frac{1}{2} + k$
(c) $\frac{x^2}{2} \left[(\log x)^2 + \frac{1}{2} \right] + k$ (d) none of these

48. Evaluate $\int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$ and the value is

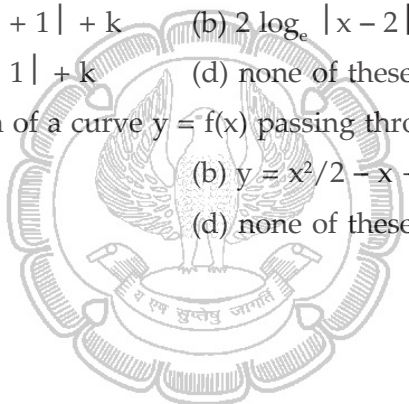
- (a) $\log_e |e^x + e^{-x}|$ (b) $\log_e |e^x + e^{-x}| + k$
(c) $\log_e |e^x - e^{-x}| + k$ (d) none of these

49. Using the method of partial fraction evaluate $\int 3x(x^2 - x - 2) dx$ and the value is equal to

- (a) $2 \log_e |x - 2| + \log_e |x + 1| + k$ (b) $2 \log_e |x - 2| - \log_e |x + 1| + k$
(c) $\log_e |x - 2| + \log_e |x + 1| + k$ (d) none of these

50. If $f'(x) = x - 1$, the equation of a curve $y = f(x)$ passing through the point (1, 0) is given by

- (a) $y = x^2 - 2x + 1$ (b) $y = x^2/2 - x + 1$
(c) $y = x^2/2 - x + 1/2$ (d) none of these





ANSWERS

Exercise 9(A)

1. a	2. b	3. c	4. d	5. c	6. a	7. d	8. b
9. d	10. b	11. c	12. a	13. d	14. a	15. b	16. c
17. a	18. c	19. a	20. a	21. b	22. a	23. d	24. b
25. c	26. a	27. b	28. c	29. a	30. b	31. c	32. a
33. b	34. a	35. a	36. b	37. b	38. a	39. c	40. a
41. b	42. c	43. a	44. b	45. a	46. b	47. c	48. a
49. c	50. a						

Exercise 9(B)

1. b	2. b	3. b	4. d	5. c	6. a	7. d	8. a
9. b	10. a	11. b	12. c	13. a	14. d	15. a	16. d
17. a	18. b	19. a	20. d	21. d	22. b	23. a	24. a
25. b	26. a	27. b	28. d	29. a	30. c	31. a	32. b
33. b	34. d	35. d	36. b	37. b	38. a	39. a	40. b
41. c	42. a	43. c	44. a	45. b	46. c	47. a	48. b
49. a	50. c						



ADDITIONAL QUESTION BANK

(A) Differential Calculus

- If $y=x^3$ then dy/dx is
(A) $x^4/4$ (B) $-x^4/4$ (C) $3x^2$ (D) $-3x^2$
- If $y=x^{2/3}$ then dy/dx is
(A) $(2/3)x^{-1/3}$ (B) $(3/5)x^{5/3}$ (C) $(-3/5)x^{5/3}$ (D) None
- If $y=x^{-8}$ then dy/dx is
(A) $-8x^{-9}$ (B) $8x^{-9}$ (C) $-8x^9$ (D) $8x^9$
- If $y=5x^2$ then dy/dx is
(A) $10x$ (B) $5x$ (C) $2x$ (D) None
- If $y = 2x^2 + x$ then dy/dx is
(A) $2(x+1)$ (B) $2(x-1)$ (C) $x + 1$ (D) $x - 1$
- If $y=4x^3-7x^4$ then dy/dx is
(A) $2x(-14x^2+6x)$ (B) $2x(14x^2-6x)$ (C) $2x(14x^2+6x)$ (D) None
- If $y=(4/3)x^3-(6/7)x^7+4x^{-3}$ then dy/dx is
(A) $4x^2-6x^6-12x^{-4}$ (B) $4x^2+6x^6-12x^{-4}$ (C) $4x^2+6x^6+12x^{-4}$ (D) None
- If $y=9x^4-7x^3+8x^2-8x^{-1}+10x^{-3}$ then dy/dx is
(A) $36x^3-21x^2+16x+8x^{-2}-30x^{-4}$ (B) $36x^3-21x^2+16x-8x^{-2}+30x^{-4}$
(C) $36x^3+21x^2+16x+8x^{-2}+30x^{-4}$ (D) None
- If $y=[(1-x)/x]^2$ then dy/dx is
(A) $2(x^{-3}+x^{-2})$ (B) $2(-x^{-3}+x^{-2})$ (C) $2(x^{-3}-x^{-2})$ (D) None
- If $y=(3x^2+1)(x^3+2x)$ then dy/dx is
(A) $15x^4+21x^2+2$ (B) $15x^3+21x^2+2$ (C) $15x^3+21x+2$ (D) None
- If $y=(3x^2+5)(2x^3+x+7)$ then dy/dx is
(A) $30x^4+39x^2+42x+5$ (B) $30x^4+39x^3+42x^2+5$
(C) $30x^4+39x^3+42x^2+5x$ (D) None
- If $y=2x^{3/2}(x^{1/2}+2)(x^{1/2}-1)$ then dy/dx is
(A) $4x+5x(x-6)^{1/2}x^{1/2}$ (B) $4x+5x(x-3)^{1/2}x^{1/2}$
(C) $4x+5x(x-2)^{1/2}x^{1/2}$ (D) None



13. If $y=(x^2-1)/(x^2+1)$ then dy/dx is
 (A) $4x(x^2+1)^{-2}$ (B) $4x(x^2+1)^2$ (C) $4x(x^2-1)^{-2}$ (D) None
14. If $y=(x+1)(2x-1)/(x-3)$ then dy/dx is
 (A) $2(x^2-6x-1)/(x-3)^2$ (B) $2(x^2+6x-1)/(x-3)^2$
 (C) $2(x^2+6x+1)/(x-3)^2$ (D) None
15. If $y=(x^{1/2}+2)/x^{1/2}$ then dy/dx is
 (A) $-x^{-3/2}$ (B) $x^{-3/2}$ (C) $x^{3/2}$ (D) None
16. If $y=(3x^2-7)^{1/2}$ then dy/dx is
 (A) $3x(3x^2-7)^{-1/2}$ (B) $6x(3x^2-7)^{-1/2}$ (C) $3x(3x^2-7)^{1/2}$ (D) None
17. If $y=(3x^3-5x^2+8)^3$ then dy/dx is
 (A) $3(3x^3-5x^2+8)^2(9x^2-10x)$ (B) $3(3x^3-5x^2+8)^2(9x^2+10x)$
 (C) $3(3x^3-5x^2+8)^2(10x^2-9x)$ (D) None
18. If $y=(6x^5-7x^3+9)^{-1/3}$ then dy/dx is
 (A) $(-1/3)(6x^5-7x^3+9)^{-4/3}(30x^4-21x^2)$ (B) $(1/3)(6x^5-7x^3+9)^{-4/3}(30x^4-21x^2)$
 (C) $(-1/3)(6x^5-7x^3+9)^{4/3}(30x^4-21x^2)$ (D) None
19. If $y=[(x^2+a^2)^{1/2}+(x^2+b^2)^{1/2}]^{-1}$ then dy/dx is
 (A) $x(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}-(x^2+b^2)^{1/2}]$
 (B) $(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}-(x^2+b^2)^{1/2}]$
 (C) $x(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}+(x^2+b^2)^{1/2}]$
 (D) $(a^2-b^2)^{-1}[(x^2+a^2)^{1/2}+(x^2+b^2)^{1/2}]$
20. If $y=\log 5x$ then dy/dx is
 (A) x^{-1} (B) x (C) $5x^{-1}$ (D) $5x$
21. If $y=x^{-1/2}$ then dy/dx is
 (A) $(-1/2)x^{-3/2}$ (B) $(1/2)x^{-3/2}$ (C) $(1/2)x^{3/2}$ (D) None
22. If $y=-3x^{-7/3}$ then dy/dx is
 (A) $7x^{-10/3}$ (B) $-7x^{-10/3}$ (C) $(-7/3)x^{-10/3}$ (D) None



23. If $y=7x^4+3x^3-9x+5$ then dy/dx is
(A) $28x^3+9(x+1)(x-1)$ (B) $28x^3+9(x+1)^2$
(C) $28x^3+9(x-1)^2$ (D) None
24. If $y=x+4x^{-1}-2x^{-7}$ then dy/dx is
(A) $1-4x^{-2}+14x^{-8}$ (B) $1+4x^{-2}-14x^{-8}$ (C) $1+4x^{-2}+14x^{-8}$ (D) None
25. If $y=(x-x^{-1})^2$ then dy/dx is
(A) $2x-2x^{-3}$ (B) $2x+2x^{-3}$ (C) $2x+2x^3$ (D) $2x-2x^3$
26. If $y=(x^{1/3}-x^{-1/3})^3$ then dy/dx is
(A) $1-x^{-2}+x^{-2/3}-x^{-4/3}$ (B) $1+x^{-2}+x^{-2/3}-x^{-4/3}$
(C) $1+x^{-2}+x^{-2/3}+x^{-4/3}$ (D) None
27. If $y=(x+a)(x+b)(x+c)$ then dy/dx is
(A) $3x^2+2ax+2bx+2cx+ab+bc+ca$ (B) $2x^2+3ax+3bx+3cx+ab+bc+ca$
(C) $3x^2+2ax+2bx+2cx+2ab+2bc+2ca$ (D) None
28. If $y=(3x^2+5x)(7x+4)^{-1}$ then dy/dx is
(A) $(21x^2+24x+20)(7x+4)^{-2}$ (B) $(21x^2+20x+24)(7x+4)^{-2}$
(C) $(21x^2+24x+4)(7x+4)^{-2}$ (D) None
29. If $y=(2x+1)(3x+1)(4x+1)^{-1}$ then dy/dx is
(A) $(24x^2+12x+1)(4x+1)^{-2}$ (B) $(24x^2+12x+3)(4x+1)^{-2}$
(C) $(24x^2+12x+5)(4x+1)^{-2}$ (D) None
30. If $y=(5x^4-6x^2-7x+8)/(5x-6)$ then dy/dx is
(A) $(75x^4-120x^3-30x^2+72x+2)(5x-6)^{-2}$
(B) $(75x^4-120x^3+30x^2-72x+2)(5x-6)^{-2}$
(C) $(75x^4-120x^3-30x^2+72x-2)(5x-6)^{-2}$ (D) None
31. If $y=(ax^2+bx+c)^{1/2}$ then dy/dx is
(A) $(1/2)(2ax+b)(ax^2+bx+c)^{-1/2}$
(B) $(-1/2)(2ax+b)(ax^2+bx+c)^{-1/2}$
(C) $(1/2)(ax+2b)(ax^2+bx+c)^{-1/2}$ (D) None



32. If $y=(2x^4+3x^3-5x+6)^{-1/3}$ then dy/dx is
 (A) $(-1/3)(2x^4+3x^3-5x+6)^{-4/3} (8x^3+9x^2-5)$
 (B) $(1/3)(2x^4+3x^3-5x+6)^{-4/3} (8x^3+9x^2-5)$
 (C) $(1/3)(2x^4+3x^3-5x+6)^{4/3} (8x^3+9x^2-5)$ (D) None
33. If $y=\log[(x-1)^{1/2}-(x+1)^{1/2}]$ then dy/dx is
 (A) $(1/2)(x^2-1)^{-1/2}$ (B) $(-1/2)(x^2-1)^{-1/2}$ (C) $(1/2)(x^2-1)^{1/2}$ (D) None
34. If $y=\log\sqrt{x+\sqrt{x^2+a^2}}$ then dy/dx is
 (A) $(1/2)(x^2+a^2)^{-1/2}$ (B) $(-1/2)(x^2+a^2)^{-1/2}$
 (C) $(1/2)(x^2+a^2)^{1/2}$ (D) None
35. If $x=3at/(1+t^3)$, $y=3at^2/(1+t^3)$, then dy/dx is
 (A) $(2t-t^4)/(1-2t^3)$ (B) $(2t-t^4)/(1+2t^3)$ (C) $(2t+t^4)/(1+2t^3)$ (D) None
36. If $y=\log[e^{3x}(5x-3)^{1/3}(4x+2)^{-1/3}]$ then dy/dx is
 (A) $3+(1/3)[5/(5x-3)-4/(4x+2)]$ (B) $3-(1/3)[5/(5x-3)-4/(4x+2)]$
 (C) $3+(1/3)[5/(5x-3)+4/(4x+2)]$ (D) None
37. If $y=x^{x^x}$ then the value of dy/dx is
 (A) $x^{x^x} [x^{x-1}+\log x \cdot x^x(1+\log x)]$ (B) $x^{x^x} [x^{x-1}+\log x \cdot (1+\log x)]$
 (C) $x^{x^x} [x^{x-1}+\log x \cdot x^x(1-\log x)]$ (D) $x^{x^x} [x^{x-1}+\log x \cdot (1-\log x)]$
38. If $x^y=e^{x-y}$ then dy/dx is
 (A) $\log x/(1-\log x)^2$ (B) $\log x/(1+\log x)^2$ (C) $\log x/(1-\log x)$ (D) $\log x/(1+\log x)$
39. If $y=(x+a)(x+b)(x+c)(x+d)/(x-a)(x-b)(x-c)(x-d)$ then the value of dy/dx is
 (A) $(x+a)^{-1}+(x+b)^{-1}+(x+c)^{-1}+(x+d)^{-1}-(x-a)^{-1}-(x-b)^{-1}-(x-c)^{-1}-(x-d)^{-1}$
 (B) $(x+a)^{-1}-(x+b)^{-1}+(x+c)^{-1}-(x+d)^{-1}+(x-a)^{-1}-(x-b)^{-1}+(x-c)^{-1}-(x-d)^{-1}$
 (C) $(x-a)^{-1}+(x-b)^{-1}+(x-c)^{-1}+(x-d)^{-1}-(x+a)^{-1}-(x+b)^{-1}-(x+c)^{-1}-(x+d)^{-1}$
 (D) None
40. If $y=x(x^2-4a^2)^{1/2}(x^2-a^2)$ then dy/dx is
 (A) $(x^4-2a^2x^2+4a^4)(x^2-a^2)^{-3/2}(x^2-4a^2)^{-1/2}$



- (B) $(x^4+2a^2x^2-4a^4)(x^2-a^2)^{-3/2}(x^2-4a^2)^{-1/2}$
(C) $(x^4+2a^2x^2+4a^4)(x^2-a^2)^{-3/2}(x^2-4a^2)^{-1/2}$ (D) None
41. If $y=(2-x)(3-x)^{1/2}(1+x)^{-1/2}$ then the value of $[dy/dx]/y$ is
(A) $(x-2)^{-1}+(1/2)(x-3)^{-1}-(1/2)(1+x)^{-1}$
(B) $(x-2)^{-1}+(x-3)^{-1}-(1+x)^{-1}$
(C) $(x-2)^{-1}-(1/2)(x-3)^{-1}+(1/2)(1+x)^{-1}$ (D) None
42. If $y=\log\{e^x\{x-2\}/(x+3)\}^{3/4}$ then dy/dx is
(A) $1+(3/4)(x-2)^{-1}-(3/4)(x+3)^{-1}$ (B) $1-(3/4)(x-2)^{-1}+(3/4)(x+3)^{-1}$
(C) $1+(3/4)(x-2)^{-1}+(3/4)(x+3)^{-1}$ (D) None
43. If $y=e^{5/x}(2x^2-1)^{1/2}$ then the value of $[dy/dx]/y$ is
(A) $(2x^3-10x^2+5)x^{-2}(2x^2-1)^{-1/2}$ (B) $(2x^3-5x^2+10)x^{-2}(2x^2-1)^{-1/2}$
(C) $(2x^3+10x^2-5)x^{-2}(2x^2-1)^{-1/2}$ (D) None
44. If $y=x^2e^{5x}(3x+1)^{-1/2}(2x-1)^{-1/3}$ then the value of $[dy/dx]/y$ is
(A) $5+2x^{-1}-(3/2)(3x+1)^{-1}-(2/3)(2x-1)^{-1}$
(B) $5+2x^{-1}-(2/3)(3x+1)^{-1}-(3/2)(2x-1)^{-1}$
(C) $5+2x^{-1}-(2/3)(3x+1)^{-1}+(3/2)(2x-1)^{-1}$ (D) None
45. If $y=x^{1/2}(5-2x)^{2/3}(4-3x)^{-3/4}(7-4x)^{-4/5}$ then the value of $[dy/dx]/y$ is
(A) $(1/2)x^{-1}-(4/3)(5-2x)^{-1}+(9/4)(4-3x)^{-1}+(16/5)(7-4x)^{-1}$
(B) $(1/2)x^{-1}-(3/4)(5-2x)^{-1}+(9/4)(4-3x)^{-1}+(16/5)(7-4x)^{-1}$
(C) $(1/2)x^{-1}+(4/3)(5-2x)^{-1}+(9/4)(4-3x)^{-1}+(16/5)(7-4x)^{-1}$
(D) None
46. If $y=x^x$ then the value of $[dy/dx]/y$ is
(A) $\log x+1$ (B) $\log x-1$ (C) $\log(x+1)$ (D) None
47. If $y=(1+x)^{2x}$ then the value of $[dy/dx]/y$ is
(A) $2[x(x+1)^{-1}+\log(x+1)]$ (B) $x(x+1)^{-1}+\log(x+1)$
(C) $2[x(x+1)^{-1}-\log(x+1)]$ (D) None



48. If $y=x^{1/x}$ then the value of $[dy/dx]/y$ is
 (A) $x^{-2}(1-\log x)$ (B) $x^2(1-\log x)$ (C) $x^{-2}(1+\log x)$ (D) None
49. If $y=(x^x)^x$ then dy/dx is
 (A) $x^{x^2+1}(1+2\log x)$ (B) $x^{x^2+1}(1+\log x)$ (C) $x^{x^2+1}(1-\log x)$ (D) None
50. If $y=x^{\log x}$ then dy/dx is
 (A) $2x^{\log x-1} \cdot \log x$ (B) $x^{\log x-1} \cdot \log x$ (C) $2x^{\log x+1} \cdot \log x$ (D) None
51. If $y=x^{\log(\log x)}$ then the value of $[dy/dx]/y$ is given by
 (A) $x^{-1}[1+\log(\log x)]$ (B) $x^{-1}[1-\log(\log x)]$
 (C) $x[1+\log(\log x)]$ (D) $x[1-\log(\log x)]$
52. If $y=x^a+a^x+x^x+a^a$ a being a constant then dy/dx is
 (A) $ax^{a-1}+a^x \log a+x^x(\log x+1)$ (B) $ax^{a-1}+a^x \log a+x^x(\log x-1)$
 (C) $ax^{a-1}+a^x \log a-x^x(\log x+1)$ (D) None
53. If $x(1+y)^{1/2}+y(1+x)^{1/2}=0$ then dy/dx is
 (A) $-(1+x^2)^{-1}$ (B) $(1+x^2)^{-1}$ (C) $-(1+x^2)^{-2}$ (D) $(1+x^2)^{-2}$
54. If $x^2-y^2+3x-5y=0$ then dy/dx is
 (A) $(2x+3)(2y+5)^{-1}$ (B) $(2x+3)(2y-5)^{-1}$ (C) $(2x-3)(2y-5)^{-1}$ (D) None
55. If $x^3-xy^2+3y^2+2=0$ then dy/dx is
 (A) $(y^2-3x^2)/[2y(3-x)]$ (B) $(y^2-3x^2)/[2y(x-3)]$
 (C) $(y^2-3x^2)/[2y(3+x)]$ (D) None
56. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ then dy/dx is
 (A) $-(ax+hy+g)/(hx+by+f)$ (B) $(ax+hy+g)/(hx+by+f)$
 (C) $(ax-hy+g)/(hx-by+f)$ (D) None
57. If $y=x^{x^{\log x}}$ then dy/dx is
 (A) $y^2/[x(1-y\log x)]$ (B) $y^2/(1-y\log x)$
 (C) $y^2/[x(1+y\log x)]$ (D) $y^2/(1+y\log x)$



58. The slope of the tangent at the point (2 -2) to the curve $x^2+xy+y^2-4=0$ is given by
(A) 0 (B) 1 (C) -1 (D) None
59. If $x^2+y^2-2x=0$ then dy/dx is
(A) $(1-x)/y$ (B) $(1+x)/y$ (C) $(x-1)/y$ (D) None
60. If $x^2+3xy+y^2-4=0$ then dy/dx is
(A) $-(2x+3y)/(3x+2y)$ (B) $(2x+3y)/(3x+2y)$
(C) $-(3x+2y)/(2x+3y)$ (D) $(3x+2y)/(2x+3y)$
61. If $x^3+5x^2y+xy-5=0$ then dy/dx is
(A) $-(3x^2+10xy+y)/[x(5x+1)]$ (B) $(3x^2+10xy+y)/[x(5x+1)]$
(C) $-(3x^2+10xy+y)/[x(5x-1)]$ (D) None
62. If $(x+y)^{m+n}-x^m y^n=0$ then dy/dx is
(A) y/x (B) $-y/x$ (C) x/y (D) $-x/y$
63. Find the fourth derivative of $\log[(3x+4)^{1/2}]$
(A) $-243(3x+4)^4$ (B) $243(3x+4)^4$ (C) $-243(4x+3)^4$ (D) None
64. If $y=[x+(1+x^2)^{1/2}]^m$ then the value of the expression $(1+x^2)d^2y/dx^2+xdy/dx-m^2y$ is
(A) 0 (B) 1 (C) -1 (D) None
65. If $y=x^m e^{nx}$ then d^2y/dx^2 is
(A) $m(m-1)x^{m-2}e^{nx}+2mnx^{m-1}e^{nx}+n^2x^m e^{nx}$
(B) $m(1-m)x^{m-2}e^{nx}+2mnx^{m-1}e^{nx}+n^2x^m e^{nx}$
(C) $m(m+1)x^{m-2}e^{nx}+2mnx^{m-1}e^{nx}+n^2x^m e^{nx}$ (D) None
66. If $y=(\log x)/x$ then d^2y/dx^2 is
(A) $(2\log x-3)/x^3$ (B) $(3\log x-2)/x^3$ (C) $(2\log x+3)/x^3$ (D) None
67. If $y=ae^{mx}+be^{-mx}$ then d^2y/dx^2 is
(A) m^2y (B) my (C) $-m^2y$ (D) $-my$
68. If $y=ae^{2x}+bx e^{2x}$ where a and b are constants the value of the expression $d^2y/dx^2-4dy/dx+4y$ is _____.
(A) 0 (B) 1 (C) -1 (D) None



69. If $y=a[x+(x^2-1)^{1/2}]^n+b[x-(x^2-1)^{1/2}]^n$ the value of the expression $(x^2-y)d^2y/dx^2+xdy/dx-n^2y$ is _____.
- (A) 0 (B) 1 (C) -1 (D) None
70. If $y=(x+1)^{1/2}-(x-1)^{1/2}$ the value of the expression $(x^2-1)d^2y/dx^2+xdy/dx-y/4$ is given by
- (A) 0 (B) 1 (C) -1 (D) None
71. If $y=\log[x+(1+x^2)^{1/2}]$ the value of the expression $(x^2+1)d^2y/dx^2+xdy/dx$ is ____.
- (A) 0 (B) 1 (C) -1 (D) None
72. If $x=at^2$ and $y=2at$ then d^2y/dx^2 is
- (A) $1/(2at^3)$ (B) $-1/(2at^3)$ (C) $2at^3$ (D) None
73. If $x=(1-t)/(1+t)$ and $t=(2t)/(1+t)$ then d^2y/dx^2 is
- (A) 0 (B) 1 (C) -1 (D) None

(B) Integral Calculus

1. Integrate w.r.t x , $5x^2$
- (A) $(5/3)x^3$ (B) $(3/5)x^3$ (C) $5x$ (D) $10x$
2. Integrate w.r.t x , $(3-2x-x^4)$
- (A) $3x-x^2-x^5/5$ (B) $3x+x^2-x^5/5$ (C) $3x+x^2+x^5/5$ (D) None
3. Integrate w.r.t x , $(4x^3+3x^2-2x+5)$
- (A) $x^4+x^3-x^2+5x$ (B) $x^4-x^3+x^2-5x$ (C) $x^4+x^3-x^2+5$ (D) None
4. Integrate w.r.t x , $(x^2-1)^2$
- (A) $x^5/5-(2/3)x^3+x$ (B) $x^5/5+(2/3)x^3+x$
 (C) $x^5/5+(3/2)x^3+x$ (D) None
5. Integrate w.r.t x , $(x^{1/2}-x/2+2x^{-1/2})$
- (A) $(2/3)x^{3/2}-(1/4)x^2+4x^{1/2}$ (B) $(3/2)x^{3/2}-(1/4)x^2+4x^{1/2}$
 (C) $(2/3)x^{3/2}+(1/4)x^2+4x^{1/2}$ (D) None
6. Integrate w.r.t x , $(1-3x)(1+x)$
- (A) $x-x^2-x^3$ (B) $x-x^2+x^3$ (C) $x+x^2+x^3$ (D) None



7. Integrate w.r.t x , $(x^4+1)/x^2$
(A) $x^3/3-1/x$ (B) $1/x-x^3/3$ (C) $x^3/3+1/x$ (D) None
8. Integrate w.r.t x , $(3x^{-1}+4x^2-3x+8)$
(A) $3\log x-(4/3)x^3+(3/2)x^2-8x$ (B) $3\log x+(4/3)x^3-(3/2)x^2+8x$
(C) $3\log x+(4/3)x^3+(3/2)x^2+8x$ (D) None
9. Integrate w.r.t x , $(x-1/x)^3$
(A) $x^4/4-(3/2)x^2+3\log x+x^{-2}/2$ (B) $x^4/4+(3/2)x^2+3\log x+x^{-2}/2$
(C) $x^4/4-(2/3)x^2+3\log x+x^{-2}/2$ (D) None
10. Integrate w.r.t x , $(x^2-3x+x^{1/3}+7)x^{-1/2}$
(A) $(2/5)x^{5/2}-2x^{3/2}+(6/5)x^{5/6}-14x^{1/2}$ (B) $(5/2)x^{5/2}-2x^{3/2}+(5/6)x^{5/6}+14x^{1/2}$
(C) $(2/5)x^{5/2}+2x^{3/2}+(6/5)x^{5/6}+14x^{1/2}$ (D) None
11. Integrate w.r.t x , $(ax^2+bx^{-3}+cx^{-7})x^2$
(A) $(1/4)ax^4+b\log x-(1/4)cx^{-4}$ (B) $4ax^4+b\log x-4cx^{-4}$
(C) $(1/4)ax^4+b\log x+(1/4)cx^{-4}$ (D) None
12. Integrate w.r.t x , $x^{6/5}$
(A) $(5/11)x^{11/5}$ (B) $(11/5)x^{11/5}$ (C) $(1/5)x^{1/5}$ (D) None
13. Integrate w.r.t x , $x^{4/3}$
(A) $(3/7)x^{7/3}$ (B) $(7/3)x^{7/3}$ (C) $(1/3)x^{1/3}$ (D) None
14. Integrate w.r.t x , $x^{-1/2}$
(A) $2x^{1/2}$ (B) $(1/2)x^{1/2}$ (C) $-(3/2)x^{-3/2}$ (D) None
15. Integrate w.r.t x , $(x^{1/2}-x^{-1/2})$
(A) $(2/3)x^{3/2}-2x^{1/2}$ (B) $(3/2)x^{3/2}-(1/2)x^{1/2}$
(C) $-(1/2)x^{-1/2}-(3/2)x^{-3/2}$ (D) None
16. Integrate w.r.t x , $(7x^2-3x+8-x^{-1/2}+x^{-1}+x^{-2})$
(A) $(7/3)x^3-(3/2)x^2+8x-2x^{1/2}+\log x-x^{-1}$
(B) $(3/7)x^3-(2/3)x^2+8x-(1/2)x^{1/2}+\log x+x^{-1}$
(C) $(7/3)x^3+(3/2)x^2+8x+2x^{1/2}+\log x+x^{-1}$ (D) None



17. Integrate w.r.t x , $x^{-1}[ax^3+bx^2+cx+d]$
 (A) $(1/3)ax^3+(1/2)bx^2+cx+d\log x$
 (B) $3ax^3+2bx^2+cx+d\log x$
 (C) $2ax+b-dx^{-2}$ (D) None
18. Integrate w.r.t x , $x^{-3}[4x^6+3x^5+2x^4+x^3+x^2+1]$
 (A) $x^4+x^3+x^2+x+\log x-(1/2)x^{-2}$
 (B) $x^4+x^3+x^2+x+\log x+(1/2)x^{-2}$
 (C) $x^4+x^3+x^2+x+\log x+2x^{-2}$ (D) None
19. Integrate w.r.t x , $[2^x+(1/2)e^{-x}+4x^{-1}-x^{-1/3}]$
 (A) $2^x/\log 2-(1/2)e^{-x}+4\log x-(3/2)x^{2/3}$
 (B) $2^x/\log 2+(1/2)e^{-x}+4\log x+(3/2)x^{2/3}$
 (C) $2^x/\log 2-2e^{-x}+4\log x-(2/3)x^{2/3}$ (D) None
20. Integrate w.r.t x , $(4x+5)^6$
 (A) $(1/28)(4x+5)^7$ (B) $(1/7)(4x+5)^7$ (C) $7(4x+5)^7$ (D) None
21. Integrate w.r.t x , $x(x^2+4)^5$
 (A) $(1/12)(x^2+4)^6$ (B) $(1/6)(x^2+4)^6$ (C) $6(x^2+4)^6$ (D) None
22. Integrate w.r.t x , $(x+a)^n$
 (A) $(x+a)^{n+1}/(n+1)$ (B) $(x+a)^n/n$ (C) $(x+a)^{n-1}/(n-1)$ (D) None
23. Integrate w.r.t x , $(x^3+2)^2 3x^2$
 (A) $(1/3)(x^3+2)^3$ (B) $3(x^3+2)^3$ (C) $3x^2(x^3+2)^3$ (D) $9x^2(x^3+2)^3$
24. Integrate w.r.t x , $(x^3+2)^{1/2}x^2$
 (A) $(2/9)(x^3+2)^{3/2}$ (B) $(2/3)(x^3+2)^{3/2}$ (C) $(9/2)(x^3+2)^{3/2}$ (D) None
25. Integrate w.r.t x , $(x^3+2)^{-3}8x^2$
 (A) $-(4/3)(x^3+2)^{-2}$ (B) $(4/3)(x^3+2)^{-2}$ (C) $(2/3)(x^3+2)^{-2}$ (D) None
26. Integrate w.r.t x , $(x^3+2)^{-1/4}x^2$
 (A) $(4/9)(x^3+2)^{3/4}$ (B) $(9/4)(x^3+2)^{3/4}$ (C) $(3/4)(x^3+2)^{3/4}$ (D) None



27. Integrate w.r.t x , $(x^2+1)^{-n} 3x$
(A) $(3/2)(x^2+1)^{1-n}/(1-n)$ (B) $(3/2)(x^2+1)^{n-1}/(1-n)$
(C) $(2/3)(x^2+1)^{1-n}/(1-n)$ (D) None
28. Integrate w.r.t x , $(x^2+1)^{-3} x^3$
(A) $-(1/4)(2x^2+1)/(x^2+1)^2$ (B) $(1/4)(2x^2+1)/(x^2+1)^2$
(C) $-(1/4)(2x^2+1)/(x^2+1)$ (D) $(1/4)(2x^2+1)/(x^2+1)$
29. Integrate w.r.t x , $1/[x \log x \log(\log x)]$
(A) $\log[\log(\log x)]$ (B) $\log(\log x)$ (C) $\log x$ (D) x^{-1}
30. Integrate w.r.t x , $1/[x(\log x)^2]$
(A) $-1/\log x$ (B) $1/\log x$ (C) $\log x$ (D) None
31. Integrate w.r.t x , $x(x^2+3)^{-2}$
(A) $-(1/2)(x^2+3)^{-1}$ (B) $(1/2)(x^2+3)^{-1}$ (C) $2(x^2+3)^{-1}$ (D) None
32. Integrate w.r.t x , $(3x+7)(2x^2+3x-2)^{-1}$
(A) $(3/4)\log(2x^2+3x-2)+(19/20)\log[(2x-1)/\{2(x+2)\}]$
(B) $(3/4)\log(2x^2+3x-2)+\log[(2x-1)/\{2(x+2)\}]$
(C) $(3/4)\log(2x^2+3x-2)+(19/20)\log[2(2x-1)(x+2)]$ (D) None
33. Integrate w.r.t x , $1/(2x^2-x-1)$
(A) $(1/3)\log[2(x-1)/(2x+1)]$ (B) $-(1/3)\log[2(x-1)/(2x+1)]$
(C) $(1/3)\log[2(1-x)/(2x+1)]$ (D) None
34. Integrate w.r.t x , $(x+1)(3+2x-x^2)^{-1}$
(A) $-(1/2)\log(3+2x-x^2)+(1/2)\log[(x+1)/(x-3)]$
(B) $(1/2)\log(3+2x-x^2)+(1/2)\log[(x+1)/(x-3)]$
(C) $-(1/2)\log(3+2x-x^2)+(1/2)\log[(x-3)/(x+1)]$ (D) None
35. Integrate w.r.t x , $(5x^2+8x+4)^{-1/2}$
(A) $(1/\sqrt{5})\log[\{\sqrt{5}x+4/\sqrt{5}+(5x^2+8x+4)^{1/2}\}]$
(B) $\sqrt{5}\log[\{\sqrt{5}x+4/\sqrt{5}+(5x^2+8x+4)^{1/2}\}]$
(C) $(1/\sqrt{5})\log[\{\sqrt{5}x+4/\sqrt{5}+(5x^2+8x+4)^{-1/2}\}]$ (D) None



36. Integrate w.r.t x , $(x+1)(5x^2+8x-4)^{-1/2}$
- (A) $(1/5)(5x^2+8x-4)^{1/2}+[1/(5\sqrt{5})]\log[5\{x+4/5+(x^2+8x/5-4/5)^{1/2}(1/6)\}]$
 (B) $(1/5)(5x^2+8x-4)^{1/2}+[1/(5\sqrt{5})]\log[5\{x+4/5+(x^2+8x/5-4/5)^{-1/2}(1/6)\}]$
 (C) $(1/5)(5x^2+8x-4)^{1/2}+[1/(5\sqrt{5})]\log[5\{x+4/5+(x^2+8x/5-4/5)^{1/2}\}]$
 (D) None
37. Integrate w.r.t x , $(x^2-1)(x^4-x^2+1)^{-1}$
- (A) $[1/(2\sqrt{3})]\log[(x^2-\sqrt{3}x+1)/(x^2+\sqrt{3}x+1)]$
 (B) $[1/(2\sqrt{3})]\log[(x^2+\sqrt{3}x+1)/(x^2-\sqrt{3}x+1)]$
 (C) $[3/(2\sqrt{3})]\log[(x^2-\sqrt{3}x+1)/(x^2+\sqrt{3}x+1)]$
 (D) None
38. Integrate w.r.t x , x^2e^{3x}
- (A) $(1/3)(x^2e^{3x})-(2/9)(xe^{3x})+(2/27)e^{3x}$
 (B) $(1/3)(x^2e^{3x})+(2/9)(xe^{3x})+(2/27)e^{3x}$
 (C) $(1/3)(x^2e^{3x})-(1/9)(xe^{3x})+(1/27)e^{3x}$ (D) None
39. Integrate w.r.t x , $\log x$
- (A) $x(\log x-1)$ (B) $x(\log x+1)$ (C) $\log x-1$ (D) $\log x+1$
40. Integrate w.r.t x , $x^n \log x$
- (A) $x^{n+1}(n+1)^{-1}[\log x-(n+1)^{-1}]$ (B) $x^{n-1}(n-1)^{-1}[\log x-(n-1)^{-1}]$
 (C) $x^{n+1}(n+1)^{-1}[\log x+(n+1)^{-1}]$ (D) None
41. Integrate w.r.t x , $xe^x(x+1)^{-2}$
- (A) $e^x(x+1)^{-1}$ (B) $e^x(x+1)^{-2}$ (C) $xe^x(x+1)^{-1}$ (D) None
42. Integrate w.r.t x , xe^x
- (A) $e^x(x-1)$ (B) $e^x(x+1)$ (C) $xe^x(x-1)$ (D) None
43. Integrate w.r.t x , x^2e^x
- (A) $e^x(x^2-2x+2)$ (B) $e^x(x^2+2x+2)$ (C) $e^x(x+2)^2$ (D) None



44. Integrate w.r.t x , $x \log x$
(A) $(1/4)x^2 \log(x^2/e)$ (B) $(1/2)x^2 \log(x^2/e)$
(C) $(1/4)x^2 \log(x/e)$ (D) None
45. Integrate w.r.t x , $(\log x)^2$
(A) $x(\log x)^2 - 2x \log x + 2x$ (B) $x(\log x)^2 + 2x \log x + 2x$
(C) $x(\log x)^2 - 2 \log x + 2x$ (D) $x(\log x)^2 + 2 \log x + 2x$
46. Integrate w.r.t x , $e^x(1+x)(2+x)^2$
(A) $e^x(2+x)^{-1}$ (B) $-e^x(2+x)^{-1}$ (C) $(1/2)e^x(2+x)^{-1}$ (D) None
47. Integrate w.r.t x , $e^x(1+x \log x)x^{-1}$
(A) $e^x \log x$ (B) $-e^x \log x$ (C) $e^x x^{-1}$ (D) None
48. Integrate w.r.t x , $x(x-1)^{-1}(2x+1)^{-1}$
(A) $(1/3)[\log(x-1) + (1/2)\log(2x+1)]$ (B) $(1/3)[\log(x-1) + \log(2x+1)]$
(C) $(1/3)[\log(x-1) - (1/2)\log(2x+1)]$ (D) None
49. Integrate w.r.t x , $(x-x^3)^{-1}$
(A) $(1/2)\log[x^2/(1-x^2)]$ (B) $(1/2)\log[x^2/(1-x)^2]$
(C) $(1/2)\log[x^2/(1+x)^2]$ (D) None
50. Integrate w.r.t x , $x^3[(x-a)(x-b)(x-c)]^{-1}$ given that
 $1/A = (a-b)(a-c)/a^3$, $1/B = (b-a)(b-c)/b^3$, $1/C = (c-a)(c-b)/c^3$
(A) $x + A \log(x-a) + B \log(x-b) + C \log(x-c)$
(B) $A \log(x-a) + B \log(x-b) + C \log(x-c)$
(C) $1 + A \log(x-a) + B \log(x-b) + C \log(x-c)$ (D) None
51. Integrate w.r.t x , $(25-x^2)^{-1}$ from lower limit 3 to upper limit 4 of x
(A) $(3/4)\log(1/5)$ (B) $(1/5)\log(3/4)$ (C) $(1/5)\log(4/3)$ (D) $(3/4)\log 5$
52. Integrate w.r.t x , $(2x+3)^{1/2}$ from lower limit 3 to upper limit 11 of x
(A) 33 (B) 100/3 (C) 98/3 (D) None



ANSWERS

(A) Differential Calculus

- | | | | | | |
|-------|-------|-------|-------|-------|-------|
| 1) C | 2) A | 3) A | 4) A | 5) A | 6) A |
| 7) A | 8) A | 9) B | 10) A | 11) A | 12) A |
| 13) A | 14) A | 15) A | 16) A | 17) A | 18) A |
| 19) A | 20) A | 21) A | 22) A | 23) A | 24) A |
| 25) A | 26) A | 27) A | 28) A | 29) A | 30) A |
| 31) A | 32) A | 33) A | 34) A | 35) A | 36) A |
| 37) A | 38) B | 39) A | 40) A | 41) A | 42) A |
| 43) A | 44) A | 45) A | 46) A | 47) A | 48) A |
| 49) A | 50) A | 51) A | 52) A | 53) A | 54) A |
| 55) A | 56) A | 57) A | 58) B | 59) A | 60) A |
| 61) A | 62) A | 63) A | 64) A | 65) A | 66) A |
| 67) A | 68) A | 69) A | 70) A | 71) A | 72) A |
| 73) A | | | | | |

(B) Integral Calculus

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|-------|-------|-------|-------|-------|-------|
| 1) A | 2) A | 3) A | 4) A | 5) A | 6) A |
| 7) A | 8) B | 9) A | 10) A | 11) A | 12) A |
| 13) A | 14) A | 15) A | 16) A | 17) A | 18) A |
| 19) A | 20) A | 21) A | 22) A | 23) A | 24) A |
| 25) A | 26) A | 27) A | 28) A | 29) A | 30) A |
| 31) A | 32) A | 33) A | 34) A | 35) A | 36) A |
| 37) A | 38) A | 39) A | 40) A | 41) A | 42) A |
| 43) A | 44) A | 45) A | 46) A | 47) A | 48) A |
| 49) A | 50) A | 51) B | 52) C | | |